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The νMSM, leptonic asymmetries, and properties of singlet fermions

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Abstract: We study in detail the mechanism of baryon and lepton asymmetry generation in the framework of the νMSM (an extension of the Standard Model by three singlet fermions with masses smaller than the electroweak scale). We elucidate the issue of CP-violation in the model and define the phase relevant for baryogenesis. We clarify the question of quantum-mechanical coherence, essential for the lepton asymmetry generation in singlet fermion oscillations and compute the relevant damping rates. The range of masses and couplings of singlet leptons which can lead to successful baryogenesis is determined. The conditions which ensure survival of primordial (existing above the electroweak temperatures) asymmetries in different leptonic numbers are analysed. We address the question whether CP-violating reactions with lepton number non-conservation can produce leptonic asymmetry *below* the sphaleron freeze-out temperature. This asymmetry, if created, leads to resonant production of dark matter sterile neutrinos. We show that the requirement that a significant lepton asymmetry be produced puts stringent constraints on the properties of a pair of nearly degenerate singlet fermions, which can be tested in accelerator experiments. In this region of parameters the νMSM provides a common mechanism for production of baryonic matter and dark matter in the universe. We analyse different fine-tunings of the model and discuss possible symmetries of the νMSM Lagrangian that can lead to them.

Keywords: Baryogenesis, Cosmology of Theories beyond the SM, Neutrino Physics, CP violation.

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Contents

1. Introduction

This paper is a continuation of the works [1] - [13] addressing the cosmological and phenomenological consequences of the νMSM. The νMSM (neutrino Minimal Standard Model) is a renormalizable extension of the Standard Model (SM) by three light singlet fermions — right-handed, or sterile neutrinos. Amazingly enough, this simple theory allows to solve in a unified way four observational problems of the SM [2, 13]. It leads to neutrino masses and thus gives rise to neutrino oscillations, absent in the SM. It provides a candidate for

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dark matter particle in the form of a long-lived sterile neutrino,[1] discussed already in [15 – 17]. It allows for baryon asymmetry generation due to coherent oscillation of the other two singlet fermions [2, 18] and electroweak anomalous fermion number non-conservation at high temperatures [19], associated with sphalerons [20]. A non-minimal coupling of the Higgs field to gravity would lead to inflation consistent with cosmological observations [13].

Let us recall the essential features of the νMSM. The lightest sterile neutrino N_1 plays the role of the dark matter particle. It should have a mass above 0.3 keV [21 – 23] (the most conservative Tremaine-Gunn bound), following from observations of rotational curves of dwarf galaxies. It is practically decoupled from other fields of the Standard Model (its Yukawa coupling to the Higgs and active neutrino must be smaller than 10^{-12}, as follows from the requirement that the mass density of the sterile neutrinos produced in the early universe does not exceed the dark matter abundance [1, 9] and from astrophysical X-ray constraints [4]). Because of the very weak coupling, it does not contribute to the mass matrix for active neutrinos [1, 2, 7] and does not play a role in baryogenesis. The two other singlet fermions, N_2 and N_3, have masses above 140 MeV (the constraint is coming from accelerator experiments combined with BBN bounds [11], see also [24]). These particles must be nearly degenerate in mass, to ensure coherent CP-violating oscillations leading to baryon asymmetry of the universe [2, 18].

In this paper we are going to study several related issues. The first one is an elaboration of the mechanism of baryogenesis in singlet fermion oscillations. Though the master equations for leptogenesis in the νMSM have already been written in [2], they were only analysed in the symmetric phase of the electroweak theory and in the regime where all singlet fermions are out of thermal equilibrium. We would like to answer the following questions. What is the CP-violating phase driving the baryogenesis? What happens with lepton asymmetry when these particles thermalize? When is the coherence of their quantum-mechanical oscillations, crucial for the resonant lepton asymmetry production lost? Are the effects of electroweak symmetry breaking essential for leptogenesis? What are the rates of non-conservation of different leptonic flavours in the νMSM? What is the time evolution of deviations from thermal equilibrium due to singlet fermions, essential for leptogenesis? Can the primordial lepton asymmetries be protected from erasure in the νMSM?

The second problem is a phenomenological one. The parameter space of the νMSM which can lead to the observed baryon asymmetry has never been explored in detail. Therefore, we would like to improve the existing cosmological constraints on masses and couplings of singlet fermions, which could be helpful for their experimental search.

The third question we address in this paper is: Can we have large lepton asymmetries *well below* the electroweak scale? The motivation for this consideration is the following. In [9] we computed the abundance of dark matter sterile neutrino in the Dodelson-Widrow (DW) scenario [15] (for earlier works see [17, 25 – 27]). This scenario assumes that

(i) no sterile neutrinos existed at temperatures above 1 GeV;

(ii) the only interactions the sterile neutrinos have are those with the ordinary neutrinos;

[1] A number of interesting astrophysical applications of keV scale sterile neutrinos can be found in [14].

(iii) the universe was (leptonic) charge symmetric at temperatures below 1 GeV.

The result was compared with two types of astrophysical bounds. The first one deals with X-ray observations of diffuse X-ray background of our and distant galaxies and Milky-Way satellites [17, 28]–[41] and gives an upper limit on the mixing angle of dark matter sterile neutrino as a function of its mass. The second bound limits the free streaming length of the dark matter particle from observation of Lyman-α clouds [42] - [46]. The prediction, even with the largest uncertainties resulting from poor knowledge of QCD dynamics at the epoch of the quark-hadron crossover, is in conflict with astrophysical bounds. This rules out the DW mechanism as a source of sterile neutrino production, if one takes for granted that the results of [44, 45] are robust. If the weaker, but more conservative Tremaine-Gunn bound is applied, then the DW mechanism can account for sterile neutrino dark matter in the universe, provided the mass of sterile neutrino is below 3.5 keV (the most conservative bound is 6 keV, see [9]).

Since there are three essential assumptions involved, these considerations force to challenge one or more of them. As was found in [6], (i) and (ii) are not valid in an extension of the νMSM by a light scalar singlet,[2] interacting with the dark matter sterile neutrino. The decays of this scalar field provide an efficient mechanism for the production of dark matter particles, which is in perfect agreement with all astrophysical constraints.

In [16] it was shown that the assumption (iii) is also crucial. Namely, Shi and Fuller (SF) demonstrated that large lepton asymmetries can boost the transitions between active and sterile neutrinos leading to a possibility of resonant creation of dark matter sterile neutrinos, satisfying both the Lyman-α and X-ray constraints even if (i) and (ii) are correct. Qualitatively, the presence of a lepton asymmetry changes the dispersion relation for active neutrinos in a way that it intersects with the dispersion relation for the sterile neutrino at some particular momentum. The level crossing leads to a transfer of the leptonic excess in active neutrinos to the sterile ones, so that the dark matter abundance is roughly proportional to the lepton asymmetry.

However, for the mechanism to work, the required lepton asymmetry must exist at temperatures $\mathcal{O}(1)$ GeV and must be much larger than the baryon asymmetry, $\frac{\Delta L}{\Delta B} \gtrsim 3 \times 10^5$, [16, 49]. At the same time, in the majority of the models with baryon and lepton number violation, proposed so far, the lepton asymmetry is of the same order of magnitude as the baryon asymmetry. The reason is that the source of lepton number violation is associated with an energy scale which is of the order or greater than the electroweak scale M_W. For example, in Grand Unified Theories the baryon and lepton numbers are broken at the scale of the order of $M_{\rm GUT} \sim 10^{15}$ GeV, in see-saw models the masses of singlet Majorana fermions are, as a rule, greater than 10^9 GeV. In the SM (or in its supersymmetric extension), the breaking of lepton and baryon numbers is related to anomaly, without any other violation terms. In these models baryogenesis takes place at temperatures $T > M_W$, and the equilibrium character of sphaleron processes ensures the relation $B = \sigma L$ [50]–[54], where σ is a coefficient of the order of one, depending on the particle content of the Standard

[2] In ref. [6] this scalar boson was playing the role of the inflaton. The same mechanism was used in a similar model for another choice of parameters in [47, 48].

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Model or its extensions.[3] A lepton asymmetry so small is irrelevant for the Big Bang Nucleosynthesis (BBN) and for dark matter sterile neutrino production. In conclusion, the existence of large lepton asymmetries seems to be very unlikely, if not impossible.

In this paper we will show that this is not necessarily the case in the νMSM. Indeed, the νMSM is very different from the models mentioned above. In particular, the energy scale of the breaking of lepton number L, existing due to Majorana neutrino masses of singlet fermions, is small (below the electroweak scale), whereas the only source for baryon number (B) violation is the electroweak chiral anomaly. We will see that these facts change the situation so that the generation of (large) leptonic asymmetries becomes possible. Basically, the baryon asymmetry of the universe is related to the lepton asymmetry at the temperature of the sphaleron freeze-out $T_{\rm EW}$, and the lepton asymmetry generation *below* $T_{\rm EW}$ leaves no trace on baryon asymmetry. The requirement that large enough lepton asymmetry is generated below the electroweak scale puts a number of stringent constraints on the properties of the singlet fermions, which can be tested in a number of accelerator experiments, discussed in [11].

Motivated by the fact that large low temperature lepton asymmetries can be a consequence of the νMSM, in an accompanying paper [49] we reanalyze the SF mechanism for production of dark matter sterile neutrinos in a charge asymmetric medium. This can be rigorously done with the use of the formalism of [8] that allows the computation of the abundance of dark matter neutrinos from first principles of statistical mechanics and quantum field theory. In particular, we find the spectra of dark matter neutrinos which can be used in warm dark matter simulations, in the subsequent Lyman-α analysis and for the study of core profiles of dwarf spheroidal galaxies. In [49] we also establish a lower bound on the leptonic asymmetry $\Delta \equiv \Delta L/L \equiv (n_L - \bar{n}_L)/(n_L + \bar{n}_L) \gtrsim 2 \times 10^{-3}$ which is needed to make the SF mechanism for sterile neutrino production consistent with X-ray and Lyman-α observations (here n_L and $\bar{n}_L$ are the number densities of leptons and anti-leptons correspondingly; other conventions to characterize the presence of a non-zero lepton asymmetry are described in appendix A of [49]). Only this result from [49] will be used in the present paper.

Our findings concerning the parameter-space of the νMSM, leading to correct baryon asymmetry and dark matter abundance, leads us to the fourth question we address in this paper. Namely, we will identify different fine-tunings between the parameters of the model, required for its phenomenological success, and discuss their possible origins.

Throughout the paper we will *assume* the validity of the standard Big Bang theory below temperatures of order 1 TeV and that the only relevant degrees of freedom are those of the νMSM (i.e. of the Standard Model plus three singlet fermions). After all, one of the strong motivations for considering the νMSM as a theory providing the physics beyond the SM is the possibility to explain neutrino oscillations, dark matter, inflation and baryon asymmetry in the framework of a minimal model, and the creation of a baryon asymmetry requires the presence of temperatures above the electroweak scale. We also *assume* that

[3] A breakdown of the relation $L \simeq B$ may happen in Affleck-Dine baryogenesis [55], if it takes place below the electroweak scale and if because of some reason the decay of squark-slepton condensate produces considerably more leptons than quarks.

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at temperatures well above the electroweak scale the concentrations of *all* singlet fermions were zero and that the universe was lepton and baryon charge symmetric at this time. This type of initial conditions may arise in the νMSM where inflaton is associated with the SM Higgs field [13]. Note also that a number of calculations in this work are on the level of approximate estimates which are valid within a factor of a few and thus may be refined. However, since even this analysis happened to be rather involved, we prefer to postpone the detailed study until the νMSM gains some direct experimental support.

The paper is organised as follows. In section 2 we review the Lagrangian of the νMSM and fix the notation. We also discuss different contributions to the mass difference of singlet fermions, essential for baryogenesis and formulate several possible scenarios for its value. In section 3 we analyse the structure of CP-violation in the model and identify the the CP-violating phase that drives baryogenesis. In section 4 we set up the master equations for analysis of kinetics of leptogenesis. In section 5 we analyse CP-odd deviations from thermal equilibrium and in section 6 CP-even perturbations. In section 7 we examine in the mechanism of leptogenesis via singlet fermion oscillations. We derive constraints on the masses and couplings of neutral leptons from the requirement that the produced baryon asymmetry has the observed value and analyse the question whether large lepton asymmetries, which can boost the dark matter production, can be generated below the electroweak scale. We also determine the parameters of the model which allow for the survival of primordial lepton asymmetries to low temperatures and are consistent with the observed baryon asymmetry. In section 8 we discuss the fine-tunings and possible symmetries of the νMSM and speculate on the origin of the νMSM Lagrangian. Section 9 is conclusions, where we summarize the results.

2. The νMSM and constraints on its parameters

For our aim it is convenient to use the Lagrangian of the νMSM in the parametrisation of ref. [7]:

$$\begin{aligned} \mathcal{L}_{\nu\text{MSM}} &= \mathcal{L}_0 + \Delta\mathcal{L}\,, \\ \mathcal{L}_0 &= \mathcal{L}_{\text{SM}} + \bar{N}_I i\partial_\mu\gamma^\mu N_I - (h_{\alpha 2}\,\bar{L}_\alpha N_2\tilde{\Phi} + M\bar{N_2}^c N_3 + \text{h.c.}), \\ \Delta\mathcal{L} &= -h_{\alpha 3}\,\bar{L}_\alpha N_3\tilde{\Phi} - h_{\alpha 1}\,\bar{L}_\alpha N_1\tilde{\Phi} - \frac{\Delta M_{IJ}}{2}\,\bar{N_I}^c N_J + \text{h.c.}\,, \end{aligned} \tag{2.1}$$

where N_I are the right-handed singlet leptons, Φ and L_α ($\alpha = e, \mu, \tau$) are the Higgs and lepton doublets respectively, h is a matrix of Yukawa coupling constants, M is the common mass of two heavy neutral fermions, different elements of $\Delta M_{IJ} \ll M$ provide a mass to the lightest sterile neutrino N_1, responsible for dark matter ($M_1 \simeq \Delta M_{11}$) and produce the small splitting of the masses of N_2 and N_3, $\Delta M_{23} = 0$, $\tilde{\Phi}_i = \epsilon_{ij}\Phi_j^*$, and M is taken to be real.

Another parametrization of the same Lagrangian is related to the mass basis of Majorana neutrinos. We write

$$\mathcal{M} = U\lambda U^T\,, \tag{2.2}$$

where $\mathcal{M}_{IJ} = M\delta_{I2}\delta_{J3} + M\delta_{I3}\delta_{J2} + \Delta M_{IJ}$, λ is a diagonal matrix with positive values λ_I, and U is a unitary matrix. The values λ_I^2 are nothing but eigenvalues of the hermitean matrix $\mathcal{M}\mathcal{M}^\dagger$ which can be diagonalized with the help of U, $\lambda_1 \approx M_1$.

Yet another possibility is to use the basis in which the matrix of Yukawa couplings $h_{\alpha I}$ is diagonal,

$$h = \tilde{K}_L f_d \tilde{K}_R^\dagger \tag{2.3}$$

with $f_d = \mathrm{diag}(f_1, f_2, f_3)$. Definitions of the matrices $\tilde{K}_L$ and $\tilde{K}_R$ can be found in [2].

The Yukawa coupling constants of the dark matter neutrino N_1 are strongly bounded by cosmological considerations [1] and by X-ray observations [4, 31]: $\sum_{\alpha,I} |h_{\alpha I} U_{I1}|^2 \lesssim 10^{-24}$. As has been demonstrated in [1], the contribution of the dark matter sterile neutrino to the masses of active neutrinos via the see-saw formula

$$[\delta M_\nu]_{\alpha\beta} = -\frac{M_{D\alpha 1} M_{D\beta 1}}{M_1} \tag{2.4}$$

is much smaller than the solar neutrino mass difference, and can thus safely be neglected (here $M_{D\alpha 1} = h_{\alpha 1} v$ and $v = 174\,\mathrm{GeV}$ is the vacuum expectation value of the Higgs field). Therefore, we set $h_{\alpha 1} = 0$ in the following and omit N_1 from the Lagrangian. Note that eq. (2.4) implies that the mass of one of the active neutrinos is much smaller [1] than the solar mass difference $\sim 0.01\,\mathrm{eV}$ and can be put to zero in what follows.

The values of the Yukawa coupling constants $h_{\alpha 2}$ and $h_{\alpha 3}$ are further constrained by the requirement that the νMSM must describe the observed pattern of neutrino masses and mixings. The following relation must hold:

$$[M_\nu]_{\alpha\beta} = -h_{\alpha I} h_{\beta J} \left[\frac{v^2}{\mathcal{M}_N}\right]_{IJ}, \tag{2.5}$$

where M_ν is the mass matrix of active neutrinos, and we denoted by $\mathcal{M}_N$ the 2×2 mass matrix of the second and third singlet fermion, $[\mathcal{M}_N]_{IJ} = \mathcal{M}_{IJ}$ for $I, J = 2, 3$. This formula can be simplified further by noting that N_2, N_3 must be highly degenerate in mass in order to ensure successful baryogenesis [2, 18]. In fact, any non-zero mass difference that still remains (it will be discussed later) is inessential for discussion of masses and mixings of active neutrinos and can be ignored [7]. We have, therefore,

$$[M_\nu]_{\alpha\beta} = -\frac{v^2}{M}\left(h_{\alpha 2} h_{\beta 3} + h_{\alpha 3} h_{\beta 2}\right) . \tag{2.6}$$

As was shown in ref. [7], this simplified situation allows to determine Yukawa coupling constants from the mass matrix of active neutrinos up to rescaling $h_{\alpha 2} \to h_{\alpha 2}/z$, $h_{\alpha 3} \to z h_{\alpha 3}$, where z is an arbitrary complex number. In addition, one can solve for the active neutrino masses explicitly:

$$m \in \{0, v^2[F_2 F_3 \pm |h^\dagger h|_{23}]/M\}, \tag{2.7}$$

where $F_i^2 \equiv [h^\dagger h]_{ii}$. This leads to two qualitatively different cases, namely the "normal hierarchy", $m_1 = 0$, $m_2 = m_{\mathrm{sol}}$, $m_3 = m_{\mathrm{atm}}$, and the "inverted hierarchy", $m_1 \approx m_2 \approx$

$m_{\rm atm},\ m_3 = 0$. Here $m_{\rm sol} \equiv \sqrt{\Delta m^2_{\rm sol}},\ \ m_{\rm atm} \equiv \sqrt{\Delta m^2_{\rm atm}}$, and $\Delta m^2_{\rm sol} \simeq 8.0 \times 10^{-5}\,{\rm eV}^2$, $\Delta m^2_{\rm atm} \simeq 2.5 \times 10^{-3}\,{\rm eV}^2$ [56]. Normal hierarchy corresponds to the case $|h^\dagger h|_{23} \approx F_2 F_3$, and the inverted hierarchy to the case $|h^\dagger h|_{23} \ll F_2 F_3$. From here it follows that

$$2F_2F_3v^2/M \simeq \kappa m_{\rm atm}\,, \tag{2.8}$$

where $\kappa = 1$ (2) for normal (inverted) hierarchy, $m_{\rm atm} \approx 0.05\,{\rm eV}$. If F_3 is taken to be very small, $F \equiv F_2$ is required to be large to keep the atmospheric mass difference in the right place. The ratio of Yukawa couplings F_2 and F_3 will play an important role in what follows and is denoted by ϵ,

$$\epsilon = \frac{F_3}{F_2}\ . \tag{2.9}$$

Then

$$F^2 = \frac{\kappa M m_{\rm atm}}{2\epsilon v^2}. \tag{2.10}$$

In the limit $\epsilon \to 0,\ \Delta M_{IJ} \to 0$ the Lagrangian acquires the global leptonic U(1) symmetry [7] which guarantees the degeneracy of the N_2 and N_3 of singlet fermions (necessary for baryogenesis), absence of mass for N_1 and absence of interactions of N_1 with active fermions (providing thus an approximate description of the required parameters of the dark matter sterile neutrino). This symmetry can be made explicit by introducing the 4-component Dirac spinor $\Psi = N_2 + N_3^c$ unifying a pair of two degenerate Majorana fermions.

For the discussion of the baryon and lepton asymmetries of the universe an essential parameter is the mass difference δM between the mass eigenstates of the two heaviest neutrinos [2, 18]. Indeed, successful baryogenesis can take place provided the mass difference is small enough. In the theory defined by the Lagrangian (2.1) there are two sources for the mass difference: the first one is related to the Majorana mass matrix, and the second one is due to the Higgs vacuum expectation value and Yukawa couplings to active fermionic flavours. The mass square difference of the physical states at the leading order of perturbation theory with respect to Yukawa couplings and ΔM_{IJ} is given by

$$\delta M = \frac{|m^2|}{M}\,, \tag{2.11}$$

where

$$m^2 \equiv 2(h^\dagger h)_{23} v^2 + M(\Delta M^*_{22} + \Delta M_{33})\ . \tag{2.12}$$

The one-loop corrections to this result are of the order of $\frac{\Delta m_\nu}{16\pi^2}\frac{M^2}{v^2}$.

One can distinguish three essentially different situations, depending on the relative importance of the mass difference induced by the Higgs field and the difference associated with Majorana masses.

In the first one $\Delta\lambda = \lambda_3 - \lambda_2$ is negligibly small so that the mass difference is entirely due to the Higgs condensate. One can easily find in this case from (2.7), (2.12) that the mass difference of heavy neutrinos is the same as that of active neutrinos,

$$\delta M = \Delta m_\nu\,, \tag{2.13}$$

and is given by $\delta M = m_{\rm atm} - m_{\rm sol} \simeq 0.04\,{\rm eV}$ for the case of normal hierarchy or $\delta M \simeq \Delta m^2_{\rm sol}/2m_{\rm atm} \simeq 8 \times 10^{-4}\,{\rm eV}$ for the case of inverted. Quite amazingly, these mass differences are roughly those at which the production of baryon asymmetry is extremized, $M\delta M \simeq \frac{M_W^3}{M_{\rm Pl}}$, where $M_{\rm Pl}$ is the Planck mass, $M_{\rm Pl} = 1.22 \times 10^{19}\,{\rm GeV}$ (see below for a more detailed discussion). We will refer to this situation as **Scenario I** for singlet fermion mass difference.

The second option is when the mass differences coming from two different sources are of the same order of magnitude. We will call this choice of parameters **Scenario II**. An extreme case is an exact compensation of the two leading contributions, which would allow to have a (low temperature) mass difference be much smaller than the active neutrino mass difference. To realize this fine tuning, the following condition is required to hold:

$$\delta M \ll \Delta m_\nu \ . \tag{2.14}$$

Though this possibility may be considered to be bizarre (as it requires that contributions of seemingly different nature are exactly the same in magnitude and different in sign) it must not be discarded since the origin of different terms in the νMSM is unknown, so that eq. (2.14) could be a consequence of some underlying (Planck scale?) physics. We will call this option **Scenario IIa** and discuss this fine-tuning in more detail in section 9.

Yet another possibility is when $\Delta\lambda \gg \Delta m_\nu$ so that the mass difference is entirely due to the Majorana masses. We will refer to this possibility as **Scenario III**.

For all three cases the mass eigenstates are related to the fields $N_{2,3}$ by a rotation with the maximal angle $\pi/4$ (up to complex phases and corrections $\sim \mathcal{O}(M_D/M)$).

3. The structure of CP violation in the νMSM

The generic Lagrangian (2.1) contains a number of new physical parameters (18) in comparison with the Standard Model. They can be counted as follows: 3 Majorana masses of singlet fermions, 3 Dirac masses, 6 mixing angles and 6 CP-violating phases. As we have already mentioned, one of the singlet fermions, playing the role of dark matter, has very small Yukawa couplings, and thus is irrelevant for baryogenesis and for active neutrino mixing matrix. Thus, the number of parameters of the νMSM, responsible for physics of heavier singlet leptons is smaller. The aim of this section is to identify these parameters and to find the CP-violating phase, relevant for baryogenesis.

After dropping the dark matter sterile neutrino N_1 from the Lagrangian (2.1) the theory contains 11 new parameters in comparison with the SM. These are 2 Majorana masses, 2 Dirac masses, 4 mixing angles and 3 CP-violating phases. In principle, all these 11 physical parameters of the νMSM can be determined experimentally by the detailed study of decays of the singlet fermions. At the same time, the mass matrix of active neutrinos in this case (note that one of neutrinos is massless in this approximation) depends on 7 parameters:[4] 3 mixing angles θ_{12}, θ_{23} and θ_{13}, one Dirac phase ϕ, one Majorana phase (α in the normal hierarchy case and the combination $\zeta = (\alpha - \beta)/2$ in the inverted hierarchy)

[4]We use the notations and parametrisation of ref. [56].

and two masses ($m_2,\ m_3$ for normal hierarchy, and $m_1,\ m_2$ for inverted). So, the 11 new parameters of the νMSM can be mapped to 7 parameters of the active neutrino mass matrix plus 4 extra ones. It is convenient to select these 4 parameters as follows. The first one is the average Majorana mass of singlet fermions N_2 and N_3,

$$M = \frac{\lambda_3 + \lambda_2}{2} \ . \tag{3.1}$$

The second is related to the diagonal elements of the Majorana mass matrix of singlet fermions, which after phase transformations without loss of generality can be written as

$$\Delta M_M = \frac{\lambda_3 - \lambda_2}{2} = \Delta M_{22} = \Delta M_{33} \ . \tag{3.2}$$

The third one is the parameter ϵ defined in (2.9), and the fourth is an extra CP-violating phase η, associated with it (see below).

With these notations, the relevant part of the νMSM Lagrangian is:

$$\begin{aligned} L_{\rm singlet} \ = \ & \left(\frac{\kappa M m_{\rm atm}}{2v^2}\right)^{\frac{1}{2}} \left[\frac{1}{\sqrt{\epsilon e^{i\eta}}} \bar{L}_2 N_2 + \sqrt{\epsilon e^{i\eta}} \bar{L}_3 N_3\right] \tilde{\Phi} \\ & - M \bar{N_2}^c N_3 - \frac{1}{2} \Delta M_M (\bar{N_2}^c N_2 + \bar{N_3}^c N_3) + \text{h.c.}\,, \end{aligned} \tag{3.3}$$

where L_2 and L_3 are the combinations of $L_e,\ L_\mu$ and L_τ

$$L_2 = \frac{\sum_\alpha h^*_{\alpha 2} L_\alpha}{F_2}\,, \qquad L_3 = \frac{\sum_\alpha h^*_{\alpha 3} L_\alpha}{F_3} \ . \tag{3.4}$$

We stress that the relation between $L_{2,3}$ and $L_{e,\mu,\tau}$ is not unitary, in general. In fact, there are 4 different relations between $L_{2,3}$ and leptonic flavours for each type of hierarchy [7] (some of them may be equivalent to each other after phase redefinition of leptonic flavours), leading to one and the same active neutrino mixing matrix. We will present and analyse below only one of them for each hierarchy, others can be treated in a similar way.

To simplify the analysis, we consider the case (suggested by experiments) when the angle θ_{13} and deviation of θ_{23} from its maximal value, $\delta\theta_{23} = \theta_{23} - \frac{\pi}{4}$ are small. Let us introduce the notations

$$\begin{aligned} D_1 \ &= \ \delta\theta_{23} \cos\theta_{12} + \theta_{13} \sin\theta_{12} e^{i\phi}\,, \qquad D_2 = \delta\theta_{23} \cos\theta_{12} - \theta_{13} \sin\theta_{12} e^{i\phi}\,, \\ D_3 \ &= \ \delta\theta_{23} \sin\theta_{12} + \theta_{13} \cos\theta_{12} e^{i\phi}\,, \qquad D_4 = \delta\theta_{23} \sin\theta_{12} - \theta_{13} \cos\theta_{12} e^{i\phi} \ . \end{aligned} \tag{3.5}$$

Then, for the normal hierarchy case we can write:

$$\begin{aligned} L_2 \ &= \ +a_1(1 + z_1 + z_2)\frac{L_\mu - L\tau}{\sqrt{2}} + a_2(1 + z_2 - z_1) L_e + a_3 \frac{L_\mu + L\tau}{\sqrt{2}}\,, \\ L_3 \ &= \ -a_1(1 - z_1 - z_2)\frac{L_\mu - L\tau}{\sqrt{2}} - a_2(1 - z_2 + z_1) L_e + a_3 \frac{L_\mu + L\tau}{\sqrt{2}}\,, \end{aligned} \tag{3.6}$$

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where the main terms are

$$\begin{aligned} a_1 &= ie^{-i(\alpha+\phi)}\sin\rho\cos\theta_{12}\,, \\ a_2 &= ie^{-i\alpha}\sin\rho\sin\theta_{12}\,, \\ a_3 &= \cos\rho\,, \\ \tan\rho &= \sqrt{\frac{m_2}{m_3}} \simeq \left(\frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}}\right)^{\frac{1}{4}} \simeq 0.4\ . \end{aligned} \tag{3.7}$$

The corrections are given by

$$\begin{aligned} z_1 &= -iD_4^* e^{i(\alpha+\phi)}\frac{\cot\rho}{\sin 2\theta_{12}}\,, \\ z_2 &= +iD_3^* e^{i(\alpha+\phi)}\frac{\cot\rho}{\sin 2\theta_{12}} - e^{-i(\alpha+\phi)}D_1\tan\rho\ . \end{aligned} \tag{3.8}$$

For the inverted hierarchy the corresponding equations are:

$$\begin{aligned} L_2 &= +ie^{-i\phi}b_1(1+t_1^*-t_2^*)\frac{L_\mu - L\tau}{\sqrt{2}} + b_2(1+t_1+t_2)L_e + (z_3-t_3)\frac{L_\mu+L\tau}{\sqrt{2}}\,, \\ L_3 &= -ie^{-i\phi}b_2^*(1-t_1^*-t_2^*)\frac{L_\mu - L\tau}{\sqrt{2}} + b_1^*(1-t_1+t_2)L_e + (z_3+t_3)\frac{L_\mu+L\tau}{\sqrt{2}}\,, \end{aligned} \tag{3.9}$$

where the main terms are

$$\begin{aligned} b_1 &= \frac{1}{\sqrt{2}}\left[\cos\theta_{12}e^{-i\zeta} + i\sin\theta_{12}e^{+i\zeta}\right]\quad, \\ b_2 &= \frac{1}{\sqrt{2}}\left[\cos\theta_{12}e^{+i\zeta} + i\sin\theta_{12}e^{-i\zeta}\right]\,, \end{aligned} \tag{3.10}$$

and the corrections are given by

$$\begin{aligned} t_1 &= \delta_{\rm inv}\frac{2i\cos 2\zeta\sin 2\theta_{12}}{3+\cos 4\zeta + 2\sin^2 2\zeta\cos 4\theta_{12}}\,, \\ t_2 &= \delta_{\rm inv}\left[\frac{1}{2} - \frac{1}{1+e^{-4i\zeta}\tan^2\theta_{12}}\right]\,, \\ z_3 &= D_4 e^{+i(\zeta-\phi)}, \qquad\qquad t_3 = iD_1 e^{-i(\zeta+\phi)}\,, \end{aligned} \tag{3.11}$$

where

$$\delta_{\rm inv} = \left(\frac{m_2-m_1}{m_2+m_1}\right) \simeq \frac{\Delta m^2_{\rm sol}}{4\Delta m^2_{\rm atm}} \simeq 8\times 10^{-3}\ . \tag{3.12}$$

In general, baryon asymmetry of the universe in the νMSM may depend on all three CP-violating phases described above. However, in a specific limit, when all charged lepton Yukawa couplings are the same, the baryogenesis is driven by a single phase, which we identify below. This can be done on general grounds and does not require complicated computation.

If the Yukawas in the charged sector are the same, one can choose a basis for leptonic doublets, in which interactions of singlet fermions have a simple form

$$(f_2\bar{l}_2 N_2 + f_3\bar{l}_3 N_3 + f_{23}\bar{l}_2 N_3)\tilde{\Phi}\,, \tag{3.13}$$

where $l_{1,2,3}$ are related to $L_{e,\mu,\tau}$ by some *unitary* transformation. Now, by the phase redefinition of l_2 and l_3 the constants f_2 and f_3 can be made real. Then, in this parametrization, the CP-violation effects must be proportional to the complex phase of the coupling f_{23}, they must vanish in the limit $f_{23} \to 0$. It is easy to see that

$$f_{23} = \frac{[h^\dagger h]_{23}}{F} \ . \tag{3.14}$$

Now, with the use of eqs. (3.7), (3.8) we get for the normal hierarchy:

$$f_{23} = \epsilon F e^{i\eta} \cos 2\rho \left[1 - 2i \tan\rho \left(\delta\theta_{23} \cos(\alpha+\phi) \cos\theta_{12} + \theta_{13} \cos\alpha \sin\theta_{12}\right)\right] \ . \tag{3.15}$$

The similar expression for the inverted hierarchy is obtained with the use of (3.10), (3.11):

$$f_{23} = -\epsilon F e^{i\eta} \left[\delta_{\rm inv} + \frac{1}{2}\left(t_3^* - z_3^*\right)\left(t_3 + z_3\right)\right] \ . \tag{3.16}$$

Yet another parameter in (3.3) which is important for the issue of CP-violation is the mass splitting ΔM_M. Indeed, in the limit $\Delta M_M = 0$ (the extreme case of **Scenario I**), the Lagrangian (3.13) acquires the global U(1) symmetry, so that the phases of $N_{2,3}$ cannot be fixed anymore by the mass terms, and the CP phase of f_{23} in the interaction (3.13) can be rotated away. This means that the measure of CP-violation, relevant for baryogenesis for the case when all charged lepton Yukawa couplings are the same, can be conveniently parametrised as

$$\delta^0_{\rm CP} = \epsilon \sin(\arg f_{23}) \sin\theta \, , \tag{3.17}$$

where

$$\tan\theta = \frac{\Delta M_M}{\Delta m_\nu} \ . \tag{3.18}$$

In reality the charged lepton Yukawa couplings are different, and the structure of CP-breaking relevant for baryogenesis is more involved. It has been found in [2] for the case when all reactions of singlet fermions are out of thermal equilibrium, see eq. (29) of that paper.[5] For convenience, we present it here in notations of our work, factoring out the largest Yukawa coupling F:

$$\delta_{\rm CP} = \frac{1}{F^6}\left[{\rm Im}[h^\dagger h]_{23} \sum_\alpha \left(|h_{\alpha 2}|^4 - |h_{\alpha 3}|^4\right) - \left(F_2^2 - F_3^2\right) \sum_\alpha \left(|h_{\alpha 2}|^2 + |h_{\alpha 3}|^2\right) {\rm Im}[h^*_{\alpha 2} h_{\alpha 3}]\right] \ . \tag{3.19}$$

With the use of relations (3.4), (3.6), (3.9), the expression (3.19) can be rewritten through the parameters of the neutrino mixing matrix, the value of ϵ and phase η. The corresponding relations are not very illuminating, so that we just summarize the main qualitative features of (3.19).

[5] The fact that this result does not depend on Yukawa couplings of charged fermions does not mean that they can be neglected. Indeed, the computation of baryon asymmetry takes into account that *all* reactions of the particles of the SM equilibrate, and the fact that charged Yukawas are non-zero and different is essential for kinetic description of the system with the use of eqs. (4.3), (4.4), (4.5).

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(i) The sign of baryon asymmetry cannot be found even if the active neutrino mixing matrix is completely known.

(ii) For small ϵ, $\delta_{\rm CP} \propto \epsilon$, similar to eq. (3.17).

(iii) Baryon asymmetry is non-zero even if $\theta_{13} = 0$ and $\delta\theta_{23} = 0$. In other words, the details of the active neutrino mass matrix have little influence on baryogenesis, if these angles are small.

(iv) Baryon asymmetry does not vanish even if $\tan\theta = 0$, i.e. in the **Scenario I** for singlet fermion mass differences. In this case the result is determined by the parameters of the neutrino mixing matrix only.

(v) In general, $\delta_{\rm CP} \neq 0$ for $\epsilon = 1$.

(vi) For inverted hierarchy case, $\delta_{\rm CP} \neq 0$ even if one takes a limit $m_1 = m_2$, $\theta_{13} = 0$, $\delta\theta_{23} = 0$.

The computation, leading to eq. (3.19), cannot be applied to the low temperature leptogenesis that occurs at temperatures in the GeV range (see section 7), because the mass of τ-lepton is comparable with a relevant temperature and with the mass of singlet leptons $N_{2,3}$. In this case the charged lepton masses certainly cannot be neglected, and the structure of CP-violation is even richer than that above the electroweak temperature. Still, CP-violating effects are proportional to ϵ, since in the limit $\epsilon \to 0$ all couplings in (3.3) can be made real.

For numerical estimates of CP-violating effects in the paper we will assume that the relevant CP-violating phase is of the order of one, so that the effects are suppresses by ϵ, writing explicitly this factor in the formulas.

4. Lepton asymmetry generation: review of theoretical framework

The detailed description of the system of singlet leptons and active fermions in the early universe is necessarily quite complicated. The number of relevant zero-temperature degrees of freedom (3 active and 3 sterile neutrinos and their antiparticles) is large,[6] and the time-scales of different processes can vary by many orders of magnitude. Moreover, due to the smallness of the sterile-active Yukawa couplings the processes with singlet fermions have in general a coherent character, making the approach based on Boltzmann equation for particle concentrations useless.

Probably, the simplest way to deal with coherent effects is to use the equation for the density matrix [57, 58, 18, 2]. In our case this is a 12×12 matrix ($12 = 3 \times 2 \times 2$ degrees of freedom for all active and sterile neutrino states), satisfying the kinetic equation (11) of [2]:

$$i\frac{d\rho}{dt} = [H, \rho] - \frac{i}{2}\{\Gamma, \rho\} + \frac{i}{2}\{\Gamma^p, 1 - \rho\}\,, \tag{4.1}$$

[6] In fact, the number of types of particle excitations in high temperature plasma is even higher, but we will assume in this paper that only zero-temperature degrees of freedom are relevant.

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where $H = p(t) + H_0 + H_{\rm int}$ is the Hermitean effective Hamiltonian incorporating the medium effects on neutrino propagation, $p(t)$ is the neutrino momentum, with $\langle p(t)\rangle \sim 3T$ (we will assume that all the neutral fermion masses are much smaller than the temperature), $H_0 = \frac{M^2}{2p(t)}$ (we include $\Delta M_{\rm IJ}$ to $H_{\rm int}$), Γ and Γ^p are the Hermitean matrices associated with destruction and production rates correspondingly, and $[\,,\,]$ $(\{\,,\,\})$ corresponds to the commutator (anti-commutator).[7] Following refs. [18, 2] we will use the Boltzmann statistics for estimates and replace the last term in (4.1) by $i\Gamma^p$. Also, following [2] we will replace Γ^p by $\frac{1}{2}\{\Gamma, \rho^{\rm eq}\}$ with $\rho^{\rm eq} = \exp(-p/T)$ being an equilibrium diagonal density matrix, ensuring the correct approach to thermal equilibrium. After these substitutions the kinetic equation takes a simple form

$$i\frac{d\rho}{dt} = [H, \rho] - \frac{i}{2}\{\Gamma, \rho - \rho^{\rm eq}\}\ . \tag{4.2}$$

This is a relaxation time approximation for the density matrix, fairly standard one in non-equilibrium statistical physics.

This equation can be simplified even further (for details see [2]) accounting for the following facts:

(i) The rates of interactions between active neutrinos are much higher that the rate of the universe expansion. Therefore, coherent effects for active neutrinos are not essential and the part of the general density matrix ρ related to active leptonic flavours can be replaced by equilibrium concentrations characterised by 3 dimensionless chemical potentials μ_α (the ordinary chemical potential divided by the temperature) giving the leptonic asymmetry in each flavour.

(ii) Active neutrinos get temperature dependent masses that are quite different from those of singlet fermions. Therefore, all non-diagonal elements of the density matrix involving simultaneously the active and sterile states can be put to zero.

(iii) The coupling of the dark matter neutrino is so weak that it decouples from the system.

This leaves us with the 2×2 density matrix ρ_N for singlet fermions N_2 and N_3, charge conjugated density matrix $\bar\rho_N$ for corresponding antiparticles (or, to be more precise, opposite chirality states), and 3 chemical potentials μ_α. The corresponding equations can be written as [2]:

$$i\frac{d\rho_N}{dt} = [H, \rho_N] - \frac{i}{2}\{\Gamma_N, \rho_N - \rho^{\rm eq}\} + i\mu_\alpha\tilde\Gamma^\alpha_N\,, \tag{4.3}$$

$$i\frac{d\bar\rho_N}{dt} = [H^*, \bar\rho_N] - \frac{i}{2}\{\Gamma^*_N, \bar\rho_N - \rho^{\rm eq}\} - i\mu_\alpha\tilde\Gamma^{\alpha *}_N\,, \tag{4.4}$$

$$i\frac{d\mu_\alpha}{dt} = -i\Gamma^\alpha_L\mu_\alpha + i{\rm Tr}\left[\tilde\Gamma^\alpha_L(\rho_N - \rho^{\rm eq})\right] - i{\rm Tr}\left[\tilde\Gamma^{\alpha *}_L(\bar\rho_N - \rho^{\rm eq})\right]\ . \tag{4.5}$$

[7] We stress that eq. (4.1), though it looks identical to eq. (1) of the earlier work [18], is in fact very different. In ref. [18] ρ is a 3×3 matrix, associated with singlet fermions only, whereas eq. (4.1) accounts for all leptonic degrees of freedom.

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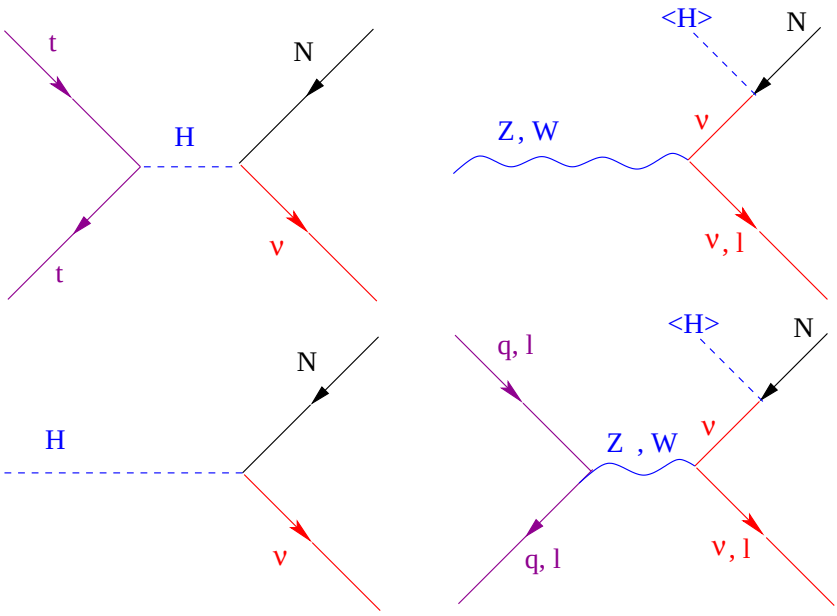

Figure 1: Diagrams for the processes which contribute to equilibration rates.

In the equation for μ_α there is no summation over α and Γ_L^α are real. The explicit expressions for the matrices describing different equilibration rates (Γ_N, $\tilde{\Gamma}_N^\alpha$, Γ_L^α, $\tilde{\Gamma}_L^\alpha$) via Yukawa coupling constants can be found in [2] for the case when the temperature is higher than the electroweak scale. They are all related to the absorptive parts of the two point functions for active or sterile neutrino states and contain a square of Yukawa couplings $h_{\alpha I}$. The real parts of the corresponding graphs together with mass squared difference between N_2 and N_3 determine the effective Hamiltonian H. For high temperatures $T \gtrsim T_{\rm EW}$ the equilibration processes are associated with Higgs, W and Z decays to singlet and active fermions, to corresponding inverse processes, and to $t\bar{t} \to N\bar{\nu}$ scattering (t is the top-quark). At smaller temperatures $T \lesssim T_{\rm EW}$ the rates are associated with W and Z exchange and singlet-active mixing through the Higgs vev, see figure 1.

In the earlier work [18] the computations and qualitative discussion were based on incomplete kinetic equations, which did not include the last terms in eqs. (4.3) and (4.4), as well as eq. (4.5). As was shown in [2], these terms are absolutely essential for the analysis of lepton asymmetry generation. Therefore, all the *quantitative* results for baryonic excess of [2, 7] and of the present paper are different from those of [18]. If some of the *qualitative* conclusions happen to be the same, we cite both papers [18] and [2] simultaneously.

The eqs. (4.3), (4.4), (4.5), supplemented by an initial condition $\rho_N = \bar{\rho}_N = \mu_\alpha = 0$ which may be fixed by inflation, provides a basis for the analysis of the lepton asymmetry generation [2]. In that paper an analytic perturbative expression for lepton asymmetry has been derived. The expression (33) of [2] is valid provided the following requirements are met:

(i) All reactions, in which the singlet fermions participate, are out of thermal equilibrium

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above the sphaleron freezing temperature. In this case a straightforward perturbation theory on Yukawa couplings of singlet fermions can be used. If this requirement is not satisfied, a perturbative expansion contains the so-called secular terms, which diverge with time and require resummations. It is one of the aims of the present paper to find out what happens if the singlet fermions equilibrate before the sphalerons decouple.

(ii) The mass difference between singlet fermions is sufficiently large so that the Higgs field contribution to it can be neglected. In other words, only **Scenario III** was considered. Naturally, we would like to extend the analysis of baryogenesis to **Scenarios I, II**, and **IIa**.

(iii) The number of oscillations of singlet fermions, related to their mass difference (see exact definition in eq. (7.14)) is much greater than one at the time of the electroweak cross-over. In this case the baryogenesis occurs in the symmetric high temperature phase of the SM and the Higgs vacuum expectation value does not play any role. We would like to understand what happens if this assumption is not satisfied, which is the case, in particular, in **Scenarios I, II**, and **IIa**.

To address all these questions we make a number of helpful transformations of kinetic equations (4.3), (4.4), (4.5). In particular, a further simplification of the system (4.3), (4.4), (4.5) can be made under assumption that the CP-violating effects are small. Let us introduce the CP-odd (ρ_-) and CP-even deviations (ρ_+) from thermal equilibrium by writing

$$\rho_N - \rho^{\rm eq} = \delta\rho_+ + \frac{\delta\rho_-}{2}\,, \quad \bar\rho_N - \rho^{\rm eq} = \delta\rho_+ - \frac{\delta\rho_-}{2}\,. \tag{4.6}$$

and neglect in (4.3), (4.4), (4.5) all terms that are of the second order in CP-odd quantities (such as $(\Gamma_N - \Gamma_N^*)\delta\rho_-$) etc. In this approximation one can decouple the equations for the CP-even deviations ρ_+ and get

$$i\frac{d\delta\rho_+}{dt} = [{\rm Re}\ H, \delta\rho_+] - \frac{i}{2}\{{\rm Re}\ \Gamma_N, \delta\rho_+\}\,, \tag{4.7}$$

with the initial condition $\delta\rho_+ = -\rho^{\rm eq}$. The equations for the CP-odd part in this approximation have the form:

$$\begin{aligned} i\frac{d\delta\rho_-}{dt} &= [{\rm Re}\ H, \delta\rho_-] - \frac{i}{2}\{{\rm Re}\ \Gamma_N, \delta\rho_-\} + i\mu_\alpha {\rm Re}\ \tilde\Gamma_N^\alpha\ + S, \\ i\frac{d\mu_\alpha}{dt} &= -i\Gamma_L^\alpha\mu_\alpha + i{\rm Tr}\left[{\rm Re}\ \tilde\Gamma_L^\alpha\delta\rho_-\right] + S_\alpha\,, \end{aligned} \tag{4.8}$$

with zero initial conditions for $\delta\rho_-$ and leptonic chemical potentials. Here the source terms S and S_μ are proportional to CP breaking parameters and given by:

$$S = 2i[{\rm Im}\ H, \delta\rho_+] + \{{\rm Im}\ \Gamma_N, \delta\rho_+\}\,, \tag{4.9}$$

$$S_\alpha = -2{\rm Tr}\left[{\rm Im}\ \tilde\Gamma_L^\alpha\delta\rho_+\right]\,. \tag{4.10}$$

They are only non-zero when CP-even deviations from thermal equilibrium exist, which is a key issue for baryogenesis and leptogenesis [59]. At the same time, if different damping rates

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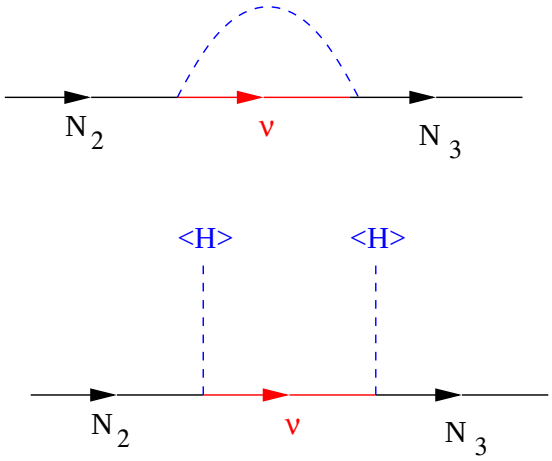

Figure 2: "Soft" contribution to the mass difference of singlet fermions coming from electroweak spontaneous symmetry breaking (lower panel) and from radiative correction (upper panel). Non-zero temperature neutrino propagator has to be used.

in (4.8) are all larger than the rate of the universe expansion after leptogenesis, the created asymmetry disappears. Therefore, to find whether baryogenesis is possible at all, one can study first the rates of different processes that equilibrate CP-odd and CP-even deviations from thermal equilibrium. This can be only skipped if *all* reactions with singlet fermions are out of thermal equilibrium, which was the case considered in [2]. If the necessary conditions for baryogenesis are found to be satisfied, an analysis of the CP-violating effects must follow.

So, we will consider first the system (4.7), (4.8) neglecting all CP-violating effects. To simplify the notations, we will take away the symbol Re of the real part from the equations.

5. CP-even deviations from thermal equilibrium

5.1 High temperature singlet fermion masses and mass eigenstates

The behaviour of the CP-even perturbations is determined by H and Γ_N, see eq. (4.7). Let us start from a discussion of the Hamiltonian $H_{\rm int}$, describing the oscillations.

The Hamiltonian $H_{\rm int}$ has the form

$$H_{\rm int} = \frac{\Delta M^2(T)}{2p}\,, \tag{5.1}$$

where $\Delta M^2(T)$ is the temperature dependent (non-diagonal) matrix of mass differences between singlet fermions. It is determined by the zero-temperature mass difference and by real parts of propagator-type graphs for sterile fermions, see figure 2.

There are two different temperature-dependent contributions to the mass difference. The bottom one is proportional to the square of the temperature dependent vacuum expectation value of the Higgs field $v(T)$ whereas the top one is proportional to T^2, coming

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from the Higgs exchange. At temperatures around and below the sphaleron freezing, interesting to us, the contribution related to the Higgs vev dominates because of the usual loop suppression and since $v(T) \gtrsim T$. We get for the high temperature case $M^2 \ll T^2$:

$$\Delta M^2(T)_{IJ} \simeq \begin{pmatrix} 0 & m^2(T) \\ m^{*2}(T) & 0 \end{pmatrix} - v^2(T) h_{\alpha I} h^*_{\alpha J} \frac{2bp}{M^2 + 2bp}\,, \tag{5.2}$$

where $m^2(T)$ is determined by eq. (2.12) with the replacement $v \to v(T)$. The function b is defined by the active neutrino propagator $1/(\not{p} + \not{\Sigma}_\alpha)$: $\not{\Sigma} = a\not{p} + b\not{u}$, where u is 4-vector of the medium. The function b in different limits is given by [60, 61]:

$$b = \begin{cases} -\frac{\pi\alpha_W T^2}{8p}\left(2 + \frac{1}{\cos^2\theta_W}\right), & T \gg M_W \\ \frac{16 G_F^2}{\pi\alpha_W}\left(2 + \cos^2\theta_W\right)\frac{7\pi^2 T^4 p}{360}, & T \ll M_W\,. \end{cases} \tag{5.3}$$

This is the so-called potential contribution to active neutrino dispersion in the medium.

From (5.2) we get for the temperature-dependent mass difference $\delta M(T)$

$$\delta M(T) \simeq \frac{v^2(T)}{2M}\sqrt{\left(F_2^2 - F_3^2\right)^2 + 4\left|(h^\dagger h)_{23} - \frac{m^2(T)}{v^2(T)}\right|^2}\,. \tag{5.4}$$

For small ϵ and high temperatures this gives in **Scenarios I, II**:

$$\delta M \simeq \frac{\kappa m_{\rm atm}}{4\epsilon}\frac{v^2(T)}{v^2}\,. \tag{5.5}$$

The temperature-dependent contribution to the mass difference is suppressed in comparison with the zero-temperature one at $2bp \ll M^2$. Taking for an estimate the typical momentum of a particle in high temperature plasma $p \simeq 3T$ one finds that this inequality is satisfied at

$$T \lesssim T_{\rm pot} = 13\left(\frac{M}{\rm GeV}\right)^{1/3}\ {\rm GeV}\,, \tag{5.6}$$

and that for these temperatures and $\epsilon \ll 1$ the mass difference is

$$\delta M(T) \simeq \sqrt{\left(\frac{\kappa m_{\rm atm}}{2\epsilon}\frac{bp}{M^2}\right)^2 + \delta M^2(0)}\,. \tag{5.7}$$

Note that depending on parameters $\delta M(T)$ can go through zero at some particular temperature, leading to level crossing and to the resonant production of lepton asymmetry, see section 7. As follows from eq. (5.4), this may only happen at $T > T_{\rm pot}$ if $\epsilon = 1$.

Let us discuss the high temperature mass eigenstates for three different scenarios of the singlet fermion mass differences (see the end of section 2 for definition).

Scenario I. In this case $\Delta\lambda = 0$ and the mass difference comes entirely from the interaction with the Higgs field.

For $\epsilon \ll 1$ the high temperature mass eigenstates N_2^T and N_3^T are close to N_2 and N_3,

$$\begin{aligned} N_2^T &\simeq \cos\beta_0 N_2 + \sin\beta_0 N_3\,, \\ N_3^T &\simeq \cos\beta_0 N_3 - \sin\beta_0 N_2\,, \end{aligned} \tag{5.8}$$

where

$$\beta_0 \simeq \frac{(h^\dagger h)_{23}}{F^2}\,, \tag{5.9}$$

whereas for $\epsilon \sim 1$ they represent the mixing of N_2 and N_3 with the angle of the order of 1. With the use of eq. (2.8) the ratio of Yukawa couplings that appear in (5.9) can be written as

$$\beta_0 \simeq \frac{\epsilon \Delta m_\nu}{\kappa m_{\rm atm}} \simeq \epsilon \begin{cases} 1 & \text{Normal hierarchy} \\ 8\times 10^{-3} & \text{Inverted hierarchy}\,. \end{cases} \tag{5.10}$$

It is not difficult to see that for temperatures below $T_{\rm pot}$ the mixing angle β gets modified,

$$\beta_0 \to \beta \simeq \beta_0 \left(1 + \frac{M^2}{2bp}\right)\,, \tag{5.11}$$

leading to (5.9) for $bp \gg M^2$ (this expression is valid provided $\beta \lesssim 1$). Therefore, the temperature T_β at which the mixing β is of the order of one is given by

$$T_\beta \simeq \beta_0^{\frac{1}{6}} T_{\rm pot} \simeq 16\ {\rm GeV} \left(\frac{\epsilon \delta M(0)}{\kappa m_{\rm atm}}\right)^{\frac{1}{6}} \left(\frac{M}{\rm GeV}\right)^{\frac{1}{3}}\,. \tag{5.12}$$

This is derived with the use of eq. (5.3) for $T \ll M_W$ and $\epsilon \ll 1$.

Scenario II. In this case both terms in eq. (5.2) have the same order of magnitude and the mixing angle β is in general of the order of one. It goes to the zero-temperature value $\pi/4$ at temperatures below T_β, see eq. (5.12). For the **Scenario IIa** at $T > T_\beta$ the mass difference is of the order of $m_{\rm atm}/\epsilon$ (see eq. (5.5)) and is much smaller than Δm_ν at lower temperatures, see eq. (5.7). Note that at $T \gtrsim T_{\rm EW}$ $v(T) \neq v$ and, therefore, $m(T) \neq 0$ even if $m(0) = 0$.

Scenario III. In this case the mass difference comes entirely from the tree Majorana mass, and the high temperature mixing angle is always close to $\pi/4$.

5.2 Damping of CP-even perturbations

Let us turn now to the part of equation (4.7) describing creation and destruction of singlet fermions. To find Γ_N we note that for initial condition $\rho = 0,\ \mu = 0$ one gets for sufficiently small times from eq. (4.7) that

$$\frac{d\rho}{dt} \simeq \Gamma_N(\mathbf{q})\,. \tag{5.13}$$

At the same time, for the time scales smaller than $1/\Gamma_N$ but much larger than microscopic time scales such as $1/T$, the derivative $\frac{d\rho}{dt}$ can be found from first principles of statistical mechanics and quantum field theory as described in [8, 9]. Therefore, we can use the methods of these papers to define the rate Γ_N.

First we note that for the computation of Γ the term $\frac{\Delta M_{IJ}}{2}\ \bar{N_I}^c N_J$ can be neglected (this term, however, must be kept in $H_{\rm int}$, as has been done above). In this case the fields N_2 and N_3 can be unified in one Dirac spinor as $\Psi = N_2 + N_3^c$. As usual, Ψ can be decomposed in creation and annihilation operators as

$$\Psi(x) = \int \frac{{\rm d}^3\mathbf{p}}{\sqrt{(2\pi)^3 2p^0}} \sum_{s=\pm} \left[\hat{a}_{\mathbf{p},s} u(\mathbf{p},s) e^{-iP\cdot x} + \hat{b}^\dagger_{\mathbf{p},s} v(\mathbf{p},s) e^{iP\cdot x}\right]\,, \tag{5.14}$$

where the spinors u, v satisfy the completeness relations

$$\sum_s u(\mathbf{p},s)\bar{u}(\mathbf{p},s) = \not{p} \ + M, \quad \sum_s v(\mathbf{p},s)\bar{v}(\mathbf{p},s) = \not{p} \ - M \ . \tag{5.15}$$

The operators $\hat{a}^\dagger_{\mathbf{q},s}$ and $\hat{b}^\dagger_{\mathbf{q},s}$ are the creation operators of a singlet fermion N_2 and an anti-fermion N_3 with momentum $\mathbf{q}$, and helicity state s. These operators are normalized as

$$\{\hat{a}_{\mathbf{p},s}, \hat{a}^\dagger_{\mathbf{q},t}\} = \delta^{(3)}(\mathbf{p}-\mathbf{q})\delta_{st} \ , \tag{5.16}$$

and V is the volume of the system. The density matrix ρ_N is associated with operators

$$\hat{\rho}_N = \frac{1}{V}\begin{pmatrix} \hat{a}^\dagger_{\mathbf{q},+}\hat{a}_{\mathbf{q},+} & \hat{a}^\dagger_{\mathbf{q},+}\hat{b}_{\mathbf{q},+} \\ \hat{b}^\dagger_{\mathbf{q},+}\hat{a}_{\mathbf{q},+} & \hat{b}^\dagger_{\mathbf{q},+}\hat{b}_{\mathbf{q},+} \end{pmatrix} \ . \tag{5.17}$$

Now, repeating literally the discussion of the section 2 of ref. [8] we arrive at

$$\Gamma^{IJ}_N(\mathbf{q}) = \frac{2n_{\rm F}(q^0)}{(2\pi)^3 2q^0}\sum_{\alpha=1}^{3} {\rm Tr}\left\{\Pi^\alpha_{IJ} a_L\Big[\rho_{\alpha\alpha}(-Q) + \rho_{\alpha\alpha}(Q)\Big]a_R\right\} , \tag{5.18}$$

where ρ is the spectral function defined in appendix B of ref. [8], and matrices Π_{IJ} are given by

$$\begin{aligned} \Pi^\alpha_{22} &= v^2(T)\left[|h_{\alpha 2}|^2 P_u(\mathbf{p}) + |h_{\alpha 3}|^2 P^c_u(-\mathbf{p})\right] , \\ \Pi^\alpha_{23} &= h_{\alpha 2}h^*_{\alpha 3} v^2(T) P_u(\mathbf{p}) , \\ \Pi^\alpha_{32} &= h^*_{\alpha 2}h_{\alpha 3} v^2(T) P^\dagger_u(\mathbf{p}) , \\ \Pi^\alpha_{33} &= v^2(T)\left[|h_{\alpha 2}|^2 P_v(\mathbf{p}) + |h_{\alpha 3}|^2 P^c_v(-\mathbf{p})\right] \ . \end{aligned} \tag{5.19}$$

The spin operators $P_u = u(\mathbf{p},+)\bar{u}(\mathbf{p},+)$ and $P_v = v(\mathbf{p},+)\bar{v}(\mathbf{p},+)$ are

$$\begin{aligned} P_{u,v} &= \frac{1}{2}(p^0+p)\left(\gamma^0 - \frac{\gamma\mathbf{p}}{p}\right)a_{L,R} + \frac{1}{2}(p^0-p)\left(\gamma^0 + \frac{\gamma\mathbf{p}}{p}\right)a_{R,L} , \\ P^c_u &= \gamma^2 P_u \gamma^2, \quad P^c_v = \gamma^2 P_v \gamma^2, \ u^c = v, \ v^c = u, \ \ p = |\mathbf{p}| \ . \end{aligned} \tag{5.20}$$

Let us discuss the structure of the matrix Γ^{IJ}_N in more detail. We will be interested in a total (integrated over momenta) rate appearing in eqs. (4.3), (4.4), (4.5):

$$\Gamma_N(T,M) = \frac{1}{T^3}\int d^3q \Gamma^{IJ}_N(\mathbf{q}) \ . \tag{5.21}$$

Then the structure of Γ_N is:

$$\Gamma_N = \frac{F^2}{F_0^2}\begin{pmatrix} R(T,M) + \epsilon^2 R_M(T,M) & \frac{(h^\dagger h)_{23}}{F^2}R(T,M) \\ \frac{(h^\dagger h)_{32}}{F^2}R(T,M) & \epsilon^2 R(T,M) + R_M(T,M) \end{pmatrix} , \tag{5.22}$$

where $F_0 = 2\times 10^{-9}$ is a convenient normalisation constant, and

$$R(T,M) = \frac{F_0^2}{F^2 T^3}\int d^3q \Gamma^{11}_N(\mathbf{q})|_{\epsilon=0} \tag{5.23}$$

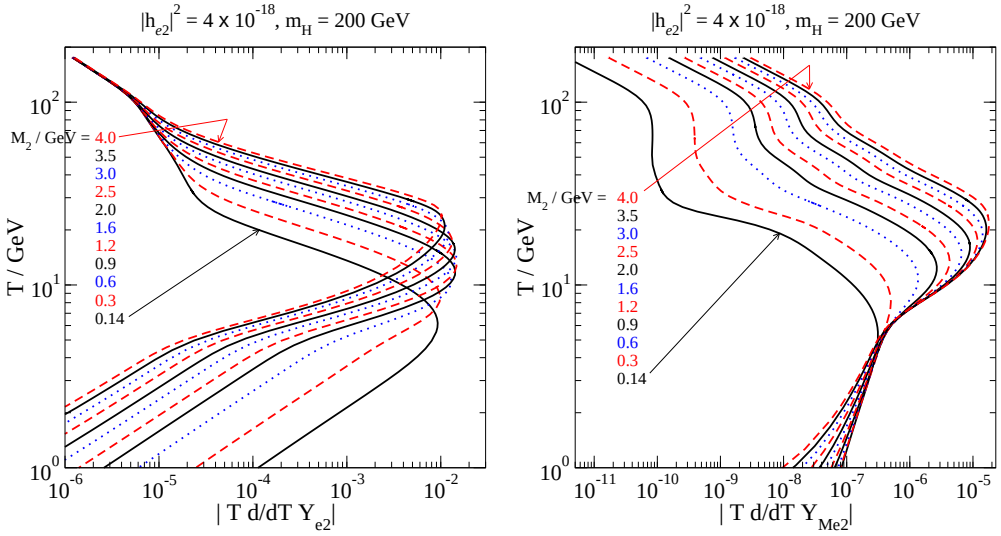

Figure 3: The temperature derivative of the yield parameter related to the rate $R(T, M)$ for the Higgs mass $m_H = 200\,\text{GeV}$, $F = F_0$ and different values of the singlet fermion mass (left panel). Right panel: the same for $R_M(T, M)$.

can be called the rate of the singlet fermion production at $F = F_0$. The quantity

$$R_M(T, M) = \frac{F_0^2}{F^2 T^3} \int d^3q \frac{q^0 - q}{q^0 + q} \Gamma_N^{11}(\mathbf{q})|_{\epsilon=0} \tag{5.24}$$

vanishes in the limit $M \to 0$ and represents the rate of the processes with violation of total lepton number (to be defined exactly below).

Computation of $R_M(T, M)$ and $R(T, M)$ is quite involved and is discussed in detail in appendix A. A large number of processes, such as W, Z and Higgs decays, together with $2 \to 2$ reactions incorporating quark and lepton initial and final states must be taken into account. The result of the computation is presented in figure 3. The vertical axis is the temperature T, and the horizontal axis is the temperature derivative of the yield parameter, defined in eq. (4.8) of [9]:

$$T\frac{dY}{dT} = -\kappa(T)R(T, M), \quad T\frac{dY_M}{dT} = -\kappa(T)R_M(T, M), \quad \kappa(T) = \frac{30M_0(T)}{4\pi^2 c_s^2(T) h_{\text{eff}}(T) T^2}\,, \tag{5.25}$$

where c_s is a speed of sound, the temperature-time relation is given by $t = \frac{M_0}{2T^2}$, $M_0 \simeq M_{\text{Pl}}/1.66\sqrt{g_{\text{eff}}}$, and the temperature dependence of the numbers of degrees of freedom g_{eff} and h_{eff} can be taken from [9]. The combination

$$\frac{1}{Y_{\text{eq}}} T\frac{dY}{dT} \tag{5.26}$$

is nothing but the ratio of the singlet fermion production rate to the Hubble constant.

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For temperatures smaller than the peak temperature the rate $R(T, M)$ can be reasonably approximated by

$$\frac{F^2}{F_0^2} R(T, M) \simeq B G_F^2 T^5 \theta_0^2 \,, \tag{5.27}$$

where $B \simeq 5$ is a numerical constant found by fitting of the numerical result, $\theta_0^2 = \frac{F^2 v^2}{M^2}$ is the zero-temperature mixing angle between the singlet fermion and active neutrinos. At the temperatures above and around the peak the suppression of the transitions due to the medium effects [15 – 17] becomes important. Also, the decays of the vector bosons and of the Higgs must be taken into account. At temperatures in the region $100 - 200\,\mathrm{GeV}$ the rate scales as $R(T, M) \propto 1/T$, while at temperatures above the peak roughly as $R(T, M) \propto 1/T^4$. In the symmetric phase of the electroweak theory, $T \gtrsim 250\,\mathrm{GeV}$, studied previously for baryogenesis via singlet fermion oscillations in [2, 18], the rate scales like $R(T, M) \propto T$.

5.3 Time evolution of CP-even perturbations

Having defined the mass matrix of singlet fermions and the matrix of the damping rates we are ready to consider the behaviour of CP-even deviations from thermal equilibrium. Let us choose the basis in which $\Delta M^2(T)_{IJ}$ is diagonal:

$$\Delta E = \begin{pmatrix} E_2 & 0 \\ 0 & E_3 \end{pmatrix} \tag{5.28}$$

and

$$\Gamma_N = \begin{pmatrix} \Gamma_{22} & \Gamma_{23} \\ \Gamma_{32} & \Gamma_{33} \end{pmatrix} \,. \tag{5.29}$$

Since for practically all temperatures $\Delta E \gg \Gamma^N_{\rm tot}$, one easily finds four different exponentials describing the time behaviour of the density matrix:

$$\exp[-((\Gamma_{22} + \Gamma_{33})/2 \pm i(E_2 - E_3))t] \,, \quad \exp(-\Gamma_{22} t) \quad \text{and} \quad \exp(-\Gamma_{33} t) \,. \tag{5.30}$$

The first one corresponds to the behaviour of the off-diagonal elements of $\Delta\rho_+$ and thus to the damping of quantum-mechanical coherence in the oscillations of singlet fermions. Two others represent the approach to thermal equilibrium of the diagonal elements of the density matrix. In general, if $\Gamma_{22} t \gtrsim 1$ and $\Gamma_{33} t \gtrsim 1$ the system equilibrates completely.

Let us consider again different scenarios for the singlet fermion mass matrix.

In the **Scenario I** the matrices $H_{\rm int}$ and $\Gamma^N_{\rm tot}$ can be simultaneously diagonalised for $T > T_{\rm pot}$ and $\epsilon \ll 1$ (up to the mass corrections M^2/T^2). Then, for $\epsilon \ll 1$ we have two very different relaxation rates for the diagonal elements of the density matrix,

$$\Gamma_{22} \simeq \frac{F^2}{F_0^2} R(T, M) \,, \quad \Gamma_{33} \simeq r_\epsilon R(T, M) + \frac{F^2}{F_0^2} R_M(T, M), \quad r_\epsilon = \frac{F^2}{F_0^2} \left(\epsilon^2 - \frac{|(h^\dagger h)_{23}|^2}{F^4} \right) \,, \tag{5.31}$$

whereas the rate of coherence loss is related to Γ_{22}. With the use of relation (2.7) the combination of Yukawa couplings which appears in (5.31) can be represented as

$$r_\epsilon \simeq \frac{\epsilon M m_{\rm atm}}{v^2 F_0^2} \begin{cases} 0.36 & \text{Normal hierarchy} \\ 1 & \text{Inverted hierarchy} \end{cases} \,, \tag{5.32}$$

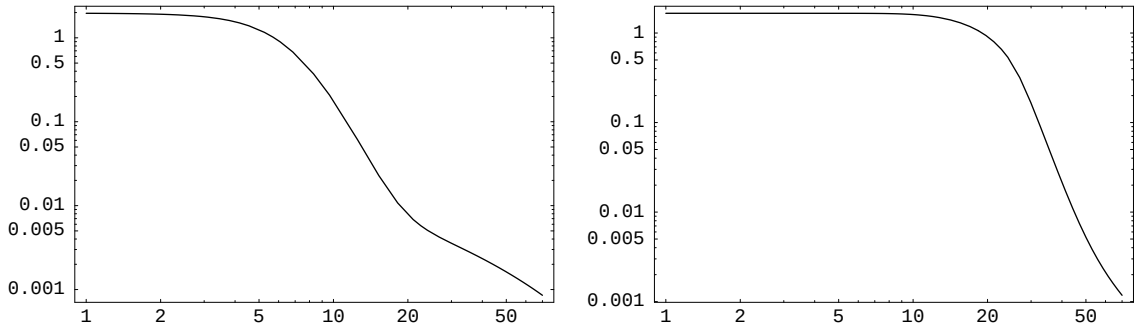

Figure 4: The ratio of the integrated rate to the equilibrium concentration of the singlet fermions for $F = F_0$ as a function of temperature (in GeV). The system enters in thermal equilibrium when this ratio is equal to one. Left panel: $M = 0.14\,\mathrm{GeV}$, right panel: $M = 4\,\mathrm{GeV}$.

leading to $\Gamma_{33}/\Gamma_{22} \propto \epsilon^2$. When the temperature falls down from $T_{\rm pot}$ to T_β the mixing angle β changes from small values $\sim \epsilon$ to $\beta \sim 1$, which modifies the (smaller) rate Γ_{33} as

$$\Gamma_{33} \to \Gamma_{33} + \sin^2\beta\,\Gamma_{22}\ . \tag{5.33}$$

As a result, at $T \lesssim T_\beta$ both rates are of the same order of magnitude and are related to the largest one Γ_{22}. Of course, for $\epsilon \sim 1$ all damping rates have the same order of magnitude for all temperatures.

In the **Scenario II** the matrix $H_{\rm int}$ can be diagonalised with the help of orthogonal transformation O, $O^T H_{\rm int} O = \mathrm{diag}$, characterised by the angle $\beta \sim 1$. In general, the rates Γ_{22} and Γ_{33} are of the same order. The same is also true for the **Scenario III** with $\Delta\lambda \gtrsim \Delta m_\nu/\epsilon$, leading to the mixing angle $\beta \simeq \pi/4$. For $\epsilon \ll 1$ all the damping rates are nearly the same and equal to $\frac{F^2}{2F_0^2} R(T, M)$. Qualitatively, if the rate of oscillations between strongly coupled singlet fermion (rate $[\Gamma_N^{\rm tot}]_{22}$) and weakly interacting fermion (rate $[\Gamma_N^{\rm tot}]_{33}$) is large, the approach to thermal equilibrium is determined by the largest rate since the system spends half of the time in the strongly interacting state. For $\Delta\lambda \lesssim \Delta m_\nu/\epsilon$ the mixing angle is between $\pi/4$ and zero, $\beta \sim \epsilon \frac{\Delta\lambda}{\Delta m_\nu}$. Varying $\Delta\lambda$ one goes smoothly from one regime to another.

We define the temperature T_+ at which the singlet fermion *enters* in thermal equilibrium from the equation

$$S_+(T_+) \equiv \frac{1}{Y_{\rm eq}(T_+)} \int_{T_+}^{\infty} \left(T \frac{dY}{dT} \right) \frac{dT}{T} = 1\,, \tag{5.34}$$

which tells that at $T = T_+$ the number of created particles is equal to the equilibrium one $Y_{\rm eq}$. If $S_+(T) \geq 1$, the initial deviations from thermal equilibrium are damped as $\exp(-S_+(T))$. The behaviour of the integrated rate $S_+(T)$ as a function of temperature is shown in figure 4.

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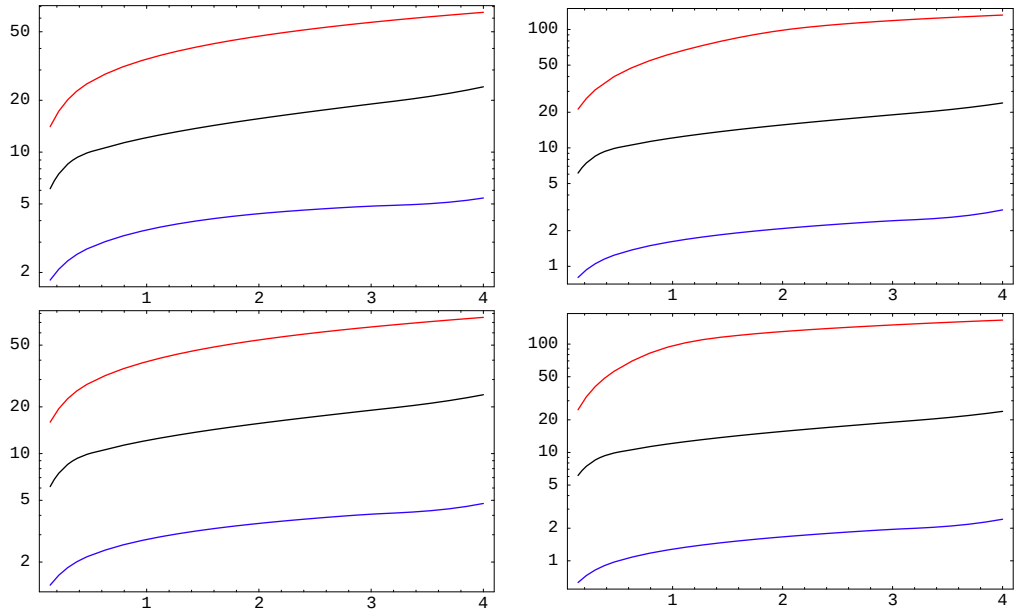

Figure 5: The temperatures (in GeV) T_+ (upper curves), T_- (lower curves) and the peak rate temperature (central curves) as a function of singlet fermion mass (in GeV). Upper panels: normal hierarchy, lower panels: inverted hierarchy. Left panels: $\epsilon = 1$, right panels: $\epsilon = 0.1$.

In full analogy, the temperature T_- at which the singlet fermions *go out* of thermal equilibrium is determined by

$$S_-(T_-) \equiv \frac{1}{Y_{\rm eq}(T_-)} \int_0^{T_-} \left(T \frac{dY}{dT} \right) \frac{dT}{T} = 1 \ . \tag{5.35}$$

We show the temperatures T_+, T_- and the temperature at which the rate is maximal in figures 5. The temperature T_+ (given roughly by $T_+ \simeq T_{\rm EW}(0.02\kappa M/\epsilon)^{\frac{1}{3}}$ at $100\,{\rm GeV} < T < 300\,{\rm GeV}$) is below the sphaleron freeze-out temperature $T_{\rm EW} \simeq 175\,{\rm GeV}$ (we take $M_H = 200\,{\rm GeV}$) for

$$\epsilon \gtrsim 0.02\kappa \frac{M}{\rm GeV} \ . \tag{5.36}$$

Looking at figures 3, 4 and 5 one can see that thermal equilibrium exists for the range of temperatures $T_- < T < T_+$. As a numerical example let us take the minimal possible mass $M = m_\pi$ and minimal value of Yukawa coupling, $F^2 \simeq 10^{-16}$ (see appendix B). It corresponds to the choice $\epsilon = 1$ and leads to $\Gamma_{22} = \Gamma_{33}$. Then the solutions to eqs. (5.34), (5.35) are $T_+ \simeq 15\,{\rm GeV}$ and $T_- \simeq 2\,{\rm GeV}$ telling that the system is in thermal equilibrium for temperatures $2\,{\rm GeV} < T < 15\,{\rm GeV}$. Asymptotically, the integrated rate approaches $S_+(T_-) \simeq 58$.

Since the Yukawa coupling chosen for this example is the minimal possible one, we reach an important conclusion that the reactions associated with the Yukawa coupling $h_{\alpha 2}$ were certainly in thermal equilibrium during some stage of the universe expansion. Moreover, in the second and third scenarios for the mass difference between singlet fermions with

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$\Delta\lambda \gtrsim \Delta m_\nu/\epsilon$ the same conclusion is valid for all elements of the density matrix $\delta\rho_+$ due to rapid oscillations between N_2 and N_3 states.

The case when $\epsilon \ll 1$ is somewhat more delicate. At first sight one may choose ϵ in such a way that the rate Γ_{33} found in (5.31) is always smaller than the rate of the universe expansion. And, indeed, the part of it, proportional to Yukawa coupling $h_{\alpha 3}$ (see (5.31)) is smaller than the Hubble rate H for all temperatures if

$$\epsilon \lesssim 3.4 \times 10^{-3} \frac{\mathrm{GeV}}{M} \begin{cases} 1 & \text{Normal hierarchy} \\ 0.36 & \text{Inverted hierarchy} \end{cases} . \tag{5.37}$$

Since the mass of singlet fermion is bounded from below by the pion mass, we get that this can only happen at $\epsilon < 2.4 \times 10^{-2}$. At the same time, the mixing angle β gets large at $T \sim T_\beta$. So, if $T_\beta > T_-$, the system equilibrates even if (5.37) is satisfied. This does not happen only if the zero temperature mass difference of singlet fermions is very small, as in **Scenario IIa**,

$$\frac{\delta M}{m_{\mathrm{atm}}} < 8 \times 10^{-5} \frac{\epsilon}{\kappa} . \tag{5.38}$$

To summarise, for *any* values of parameters, consistent with observed pattern of neutrino oscillations, with the exception of the **Scenario IIa** and for $M \ll M_W$, the CP-even deviations from thermal equilibrium are damped in some temperature interval $[T_+, T_-]$ below the electroweak scale. The ratio of the peak rate for equilibration of any element of $\delta\rho_+$ to the Hubble rate is at least 58. If the **Scenario IIa** is realized, and relations (5.37), (5.38) are satisfied, deviations from thermal equilibrium in CP-even perturbations are substantial for all temperatures. Moreover, in *any* scenario for singlet fermion mass difference, the coherence in $N_2 \leftrightarrow N_3$ oscillations is lost in the temperature interval $[T_+, T_-]$. Thus, the lepton asymmetry generation may occur either above T_+ or below T_- (see section 7 for details).

As we discussed, the CP-even deviations are important for *generation* of the lepton asymmetry. The produced asymmetry must not be diluted by reactions that can change it. Thus, we consider the CP-odd deviations from thermal equilibrium in the next subsection in order to understand whether the asymmetry that was generated before $T \simeq T_+$ or below T_- can survive the subsequent evolution.

6. CP-odd deviations from thermal equilibrium

The CP-odd deviations from thermal equilibrium are described by eq. (4.8). Having found the matrices H_{int} and Γ_N in the previous subsection we still should compute six 2×2 matrices $\tilde{\Gamma}^\alpha_L$, $\tilde{\Gamma}^\alpha_N$ and 3 rates Γ^α_L. They are coming from imaginary parts of the diagrams shown in figure 6 and have the following structure (we integrated the rates over momenta but are keeping the same notations):

$$\tilde{\Gamma}^\alpha_N \simeq \frac{1}{F_0^2} \begin{pmatrix} |h_{\alpha 2}|^2 R(T,M) - |h_{\alpha 3}|^2 R_M(T,M) & h^*_{\alpha 2} h_{\alpha 3} R(T,M) \\ h^*_{\alpha 3} h_{\alpha 2} R(T,M) & |h_{\alpha 3}|^2 R(T,M) - |h_{\alpha 2}|^2 R_M(T,M) \end{pmatrix} ,$$

$$\tilde{\Gamma}^\alpha_L \simeq \tilde{\Gamma}^\alpha_N , \tag{6.1}$$

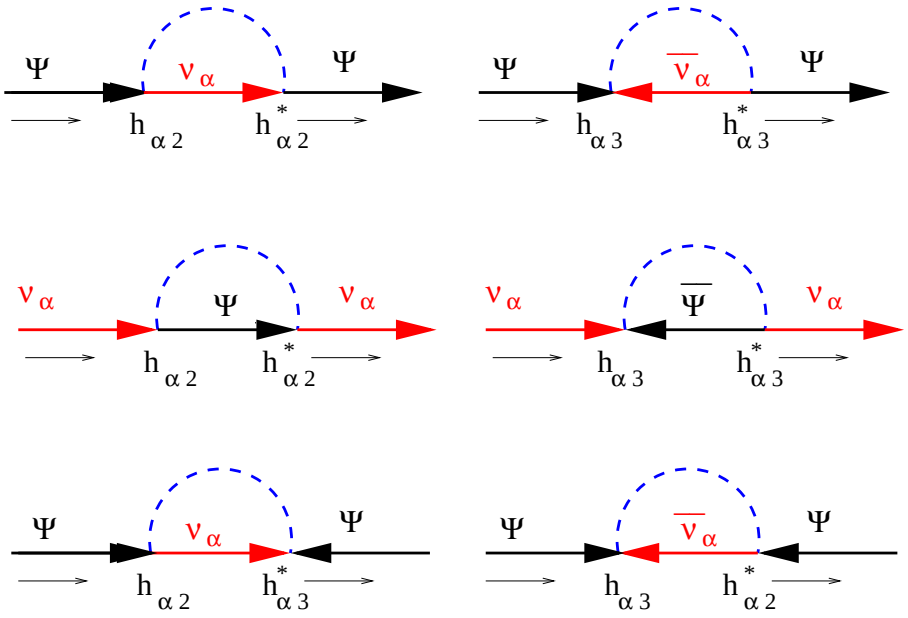

Figure 6: The propagator-type diagrams for computation of the damping rates. The Higgs line can be cut and replaced by $v(T)^2$; the active neutrino propagator contains one-loop corrections. The incoming (outcoming) fermions correspond to two arrows entering (exiting) the vertex. Outcoming antifermion corresponds to arrows in opposite directions, $\Psi = N_2 + N_3^c$.

$$\Gamma_L^\alpha \simeq \frac{1}{F_0^2}(|h_{\alpha 2}|^2 + |h_{\alpha 3}|^2)(R(T,M) + R_M(T,M)) \ .$$

The minus signs in eqs. (6.1) in front of mass corrections come about since the corresponding terms in (4.3), (4.4), (4.5) are proportional to the chemical potentials μ_α (notice the change of direction of the fermionic line in figure 6).

6.1 Approximate conservation laws and damping rates

The structure of (6.1) is almost uniquely fixed by the field-theoretical consideration presented below. Indeed, the CP-odd deviations from thermal equilibrium can be considered as average values of the densities of fermionic currents, which may be exactly conserved for some particular choice of the parameters of the νMSM.

In the limit when all Yukawa couplings and Majorana masses of singlet fermions are equal to zero the νMSM has five conserved leptonic numbers:

$$\mathcal{L}_\alpha = \int \mathrm{d}^3\mathbf{x}\, J_\alpha^0 \,, \tag{6.2}$$

where $\alpha = 1, \ldots, 5$. Three of the currents are related to the active leptonic flavours,

$$J_\alpha^\mu = \left[\bar{L}_\alpha \gamma^\mu L_\alpha + \bar{E}_\alpha \gamma^\mu E_\alpha\right] \ , \tag{6.3}$$

where E_α are the right charged leptons. The other two conserved currents count the asymmetries in singlet fermions N_2 and N_3,

$$N_2^\mu = \bar{N}_2 \gamma^\mu N_2 \,, \quad N_3^\mu = \bar{N}_3 \gamma^\mu N_3 \ . \tag{6.4}$$

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When the Yukawa couplings and Majorana masses are switched on, none of these numbers are conserved any more.

To make the discussion more transparent, consider the following combinations of the currents introduced above:

$$J_4^\mu = J_L^\mu = \sum_{\alpha=1}^{3} J_\alpha^\mu + N_2^\mu - N_3^\mu \ . \tag{6.5}$$

and

$$J_5^\mu = J_F^\mu = \sum_{\alpha=1}^{3} J_\alpha^\mu + N_2^\mu + N_3^\mu \ . \tag{6.6}$$

The first current J_L^μ (total leptonic number) corresponds precisely to the leptonic number symmetry defined in [7] which is exact in the limit $h_{\alpha 3} \to 0, \quad \Delta M_{IJ} \to 0$, whereas the second current (it can be called total fermionic number) is conserved when all Majorana neutrino masses are put to zero. What concerns the currents J_α^μ for a given α, they are conserved in the limit $h_{\alpha 3} \to 0, \ h_{\alpha 2} \to 0$.

Now, if some combination of the currents introduced above is exactly conserved, the equations (4.8) with zero source terms must have a time-independent solution for any choice of initial conditions. As an example consider first the limit $M \to 0, \ \epsilon \neq 0$. In this case the current J_5^μ is exactly conserved, and we must have

$$\frac{d}{dt}\left[\mathrm{Tr}\delta\rho_- + \sum \mu_\alpha\right] = 0 \tag{6.7}$$

for any $\delta\rho_-$ and μ_α. This leads to

$$\Gamma_N = \sum_\alpha \tilde{\Gamma}_L^\alpha \ \text{ and } \ \mathrm{Tr}\tilde{\Gamma}_N^\alpha = \Gamma_L^\alpha \ \text{ for } M = 0. \tag{6.8}$$

In the another limit $M \neq 0, \ \epsilon \to 0$ it is the current J_4^μ which is exactly conserved and

$$\frac{d}{dt}\left[\mathrm{Tr}\tau_3\delta\rho_- + \sum_\alpha \mu_\alpha\right] = 0 \tag{6.9}$$

for any $\delta\rho_-$ and μ_α (here τ_3 is the Pauli matrix). This gives

$$\Gamma_N^{22} = \sum_\alpha \tilde{\Gamma}_L^{\alpha 22}, \ \Gamma_N^{33} = -\sum_\alpha \tilde{\Gamma}_L^{\alpha 33}, \ \tilde{\Gamma}_L^{\alpha 23} = \Gamma_N^{23} = 0 \ \text{ and } \ \mathrm{Tr}\tau_3\tilde{\Gamma}_N^\alpha = \Gamma_L^\alpha \ \text{ for } \epsilon = 0. \tag{6.10}$$

In more general terms, the consistency condition can be formulated as follows. Rewrite eq. (4.8) with $S = S_\mu = 0$ in the form

$$\frac{dz}{dt} = Dz \,, \tag{6.11}$$

where z is a vector with 7 components, $z = (\delta\rho_-^{11}, \delta\rho_-^{12}, \delta\rho_-^{21}, \delta\rho_-^{22}, \mu_\alpha)$ and D is the 7×7 matrix constructed from $\tilde{\Gamma}_L^\alpha$, $\tilde{\Gamma}_N^\alpha$, Γ_L^α and H. The time-independent solution appears when D has a zero eigenvalue. Then, we must have $\det D = 0$ for the following choices of

parameters, corresponding to the conservation of the 5 currents introduced above: $h_{1I} = 0$ for $I = 2, 3$, corresponding to conservation of the leptonic number of the first generation (and similar relations for the second and third generation), $h_{\alpha 3} = 0$, corresponding to conservation of the current J^μ_4 (and an equivalent relation for $N_2 \leftrightarrow N_3$), and $M = 0$, leading to conservation of J^μ_5. One can check that eq. (6.1) indeed satisfies these requirements.

It is instructive to find the damping rates in the limit $M \to 0,\ \ \epsilon \to 0$. In this case the matrix D has two zero eigenvalues corresponding to the conservation of currents J^μ_4 and J^μ_5, 2 complex eigenvalues

$$\frac{F^2}{2F_0^2} R(T, M) \pm i(E_2 - E_3) \tag{6.12}$$

corresponding, as in the case of CP-even perturbations, to the off-diagonal elements of the density matrix $\delta\rho_-$, and three eigenvalues related to the damping rates of three different leptonic flavours,

$$\gamma_i = \frac{F^2 x_\alpha}{F_0^2} R(T, M)\,, \tag{6.13}$$

where x_i are the roots of the cubic equation

$$x^3 + 2x^2 + \frac{3}{2}\left(1 - \frac{\sum_\alpha h^4_{\alpha 2}}{F^4}\right) x + \frac{4 h^2_{e2} h^2_{\mu 2} h^2_{\tau 2}}{F^6} = 0\ . \tag{6.14}$$

If, for example, $h_{e2} \ll h_{\mu 2},\ h_{e2} \ll h_{\tau 2}$ then the smallest root of eq. (6.14) is approximately given by $5h^2_{e2}/4F^2$. From (6.12) we can see that the coherence in CP-odd perturbations is lost at the same time as it is in CP-even perturbations. As for the damping rates of active flavours, with the use of constraints (B.2), (B.3) (see appendix B) one finds that the integrated rates corresponding to γ_i

$$S_i = \frac{1}{Y_i^{\rm eq}} \int_\infty^{T_-} [\kappa(T)\gamma_i]\, \frac{dT}{T} \tag{6.15}$$

are at least

$$S_1 \simeq 8.2/\epsilon, \quad S_2 \simeq 50/\epsilon, \quad S_3 \simeq 156/\epsilon\,, \text{ Normal hierarchy}\,, \tag{6.16}$$

$$S_1 \simeq 32/\epsilon, \quad S_2 \simeq 22/\epsilon, \quad S_3 \simeq 122/\epsilon\,, \text{ Inverted hierarchy}\,, \tag{6.17}$$

where the smallest number in (6.16) corresponds to the asymmetry in the electronic flavour. Eq. (6.17) shows that if the hierarchy is inverted, all the rates exceed the rate of the universe expansion by a factor of at least 22 (corresponding to the damping of asymmetry which existed before the equilibrium period by a factor smaller than $\simeq e^{-22} \sim 3\times 10^{-10}$). For the case of the normal hierarchy eq. (6.16) shows that the damping is at least $\simeq e^{-8.2} \sim 3\times 10^{-4}$. This leads to the conclusion that the reactions which change leptonic numbers in each generation were certainly in thermal equilibrium during some time below the electroweak scale which is good enough to dilute the lepton asymmetry below the level required for resonant production of dark matter. At this point the νMSM is very different from the Standard Model, where leptonic numbers are conserved (up to electroweak anomaly).

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6.2 Protection of lepton asymmetries

The fact that the flavour changing reactions were in thermal equilibrium during some period of the universe expansion below the electroweak scale would at first sight mean that no (large) asymmetry in active leptonic flavours can exist at small temperatures. However, this conclusion is not necessarily true since some combination of asymmetries in active and *sterile* flavours may be protected from erasure due to the existence of approximate conservation laws of currents J^μ_4 and J^μ_5. The only certain thing for the moment is that the low temperature remnants of high-temperature leptonic asymmetries in active neutrinos are flavour-blind, i.e. $\mu_e \simeq \mu_\mu \simeq \mu_\tau \equiv \mu$. This fact allows to simplify the further analysis replacing the system of equations (4.8) with zero sources by

$$\begin{aligned} i\frac{d\delta\rho_-}{dt} &= [H, \delta\rho_-] - \frac{i}{2}\{\Gamma_N, \delta\rho_-\} + i\mu\sum_\alpha \tilde{\Gamma}^\alpha_N\,, \\ i\frac{d\mu}{dt} &= -i\mu\frac{1}{3}\sum_\alpha \Gamma^\alpha_L + i\mathrm{Tr}\left[\frac{1}{3}\sum_\alpha \tilde{\Gamma}^\alpha_L \delta\rho_-\right]\,. \end{aligned} \tag{6.18}$$

To consider the possibility of protection of lepton asymmetry we start from the **Scenario I** for the mass difference of singlet fermions. Then for $bp \gg M^2$ the Hamiltonian $H_{\rm int}$ can be diagonalized simultaneously with the damping rates in eq. (6.18), and one finds that for small ϵ and M the rates γ_4 and γ_5 are

$$\gamma_4 \simeq r_\epsilon R(T, M)\,, \tag{6.19}$$

$$\gamma_5 \simeq \frac{4F^2}{5F_0^2} R_M(T, M)\,, \tag{6.20}$$

where r_ϵ is defined in (5.32). In comparison with $\gamma_{1,2,3}$, the rate γ_4 is suppressed by ϵ^2 whereas the rate γ_5 is suppressed by M^2. For $bp \lesssim M^2$ the mass matrix $H_{\rm int}$ is not proportional to Γ_N, $\sum_\alpha \tilde{\Gamma}^\alpha_N$ and $\sum_\alpha \tilde{\Gamma}^\alpha_L$ any longer, leading to the rate

$$\gamma_4 \to \gamma_4 + \sin^2\beta\,\Gamma_{22}\,, \tag{6.21}$$

where Γ_{22} is defined in eq. (5.31) and the angle β in eqs. (5.9), (5.11). At the same time, the rate γ_5 is not changed. Now, repeating the considerations of the previous section one finds that J^4_μ is protected from erasure only if inequalities (5.37), (5.38) are satisfied simultaneously, i.e. only for **Scenario IIa**.

Another leptonic charge which can be protected from erasure by the processes with lepton number non-conservation is J^5_μ. If $\max\left(\frac{\gamma_5}{H}\right) \lesssim 1$ then the density matrix at low temperatures has the form

$$\rho_{\rm eq} = \exp\left(-\frac{H}{T} - \mu_5 Q_5\right) = \exp\left(-\frac{H}{T} - \mu_5(L + Q_2 + Q_3)\right)\,, \tag{6.22}$$

where μ_5 is the chemical potential corresponding to the effectively conserved charge $Q_5 = \int d^3x J^0_5$. In this case the previously generated asymmetry in Q_5 survives, and the fact that Q_5 contains the currents corresponding to active flavours ensures non-zero asymmetry

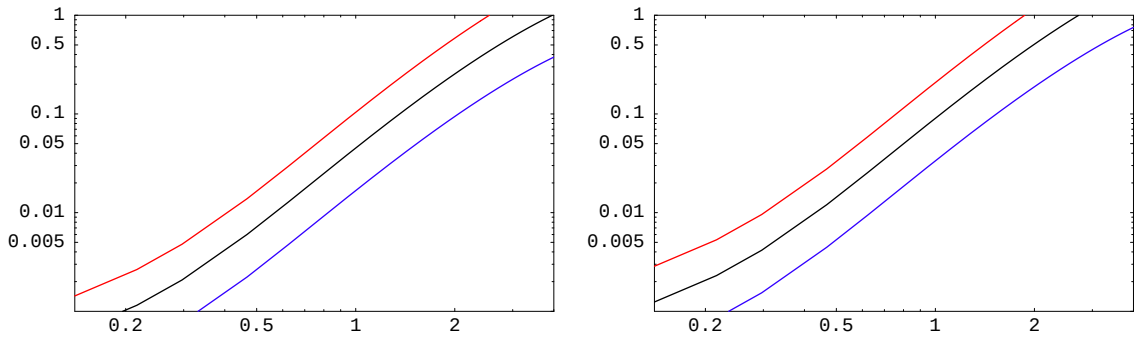

Figure 7: The region of parameters in [ϵ (vertical axis), M/GeV] plane for which the low temperature lepton asymmetry Q_5 is "protected" from erasure for normal (left panel) and inverted (right panel) hierarchies of neutrino masses. The upper curve corresponds to the damping factor e^{-1}, the lower curve to 0.002, and the middle one to 0.1.

in lepton number, which is essential for the resonant production of dark matter sterile neutrinos. Independently of the choice of parameters, the chemical potentials for $N_{2,3}$ are the same as those of the active fermions, which is the consequence of the fact that the transitions $L_\alpha \to N_{2,3}$ are in thermal equilibrium. Exactly the same conclusions are valid for the second and third scenarios for the fermionic mass difference.

The region of the parameters in which the asymmetry in Q_5 is protected can be found from the condition that the peak value of $\gamma_5/(2H)$ does not exceed 1. We plot this region in figure 7.

To summarize, the existence of a lepton asymmetry at small temperatures $\sim 100\,\mathrm{MeV}$ is only possible in the following situations:

(i) The asymmetry is produced below the temperature T_-, when the processes that damp the CP-even and CP-odd deviations go off thermal equilibrium.

(ii) The asymmetry in Q_5 is produced above T_+ and the νMSM parameters lie in the range shown in figure 7 ensuring that it is not erased later on.

(iii) The asymmetry in Q_4 is produced above T_+ and the νMSM parameters lie in the range (5.37), (5.38).

In the next section we will add to the analysis an input from the dynamics of lepton asymmetry generation which will allow to choose between these possibilities and to add further constraints.

7. Lepton asymmetry generation and constraints on masses and couplings of singlet fermions

To find the leptonic asymmetry one should solve equations (4.3), (4.4), (4.5) with zero initial conditions for chemical potentials and for the elements of the density matrices of singlet fermions. Due to the fact that the number of equations and different time scales is large

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(the equation count for real variables is as follows: 4 for ρ, 4 for $\bar{\rho}$, and 3 for μ_α) this cannot be done analytically. Nevertheless, the behaviour of the system can be understood on the qualitative level with the results of section 5 and section 6 and a number of quantitative estimates can be made.

Let us start from the small time behaviour of the system, when *all* reactions involving singlet fermions are out of thermal equilibrium, so that the largest exponential in (5.30), $\Gamma_{22}t$ is smaller than 1. This regime was considered in [2] for **Scenario III**, assuming that the number of oscillations of singlet fermions, defined below in eq. (7.14), at the time of electroweak cross-over, is much larger than one. We will generalize this analysis to a more general case, accounting for the electroweak symmetry breaking effects and considering also the time so short that $\delta M(T)t \lesssim 1$.

For these purposes it is convenient to transform the system in a form that does not contain the term responsible for oscillations between the two singlet fermion flavours, $[H, \rho_N]$, see [2]. This can be done by introducing $\tilde{\rho}_N$ related to ρ in the following way:

$$\rho_N = U(t)E(t)\tilde{\rho}_N E^\dagger(t)U^\dagger(t), \quad E(t) = \exp\left(-i\int_0^t dt'\Delta E(t')\right) , \tag{7.1}$$

where the matrix $U(t)$ converts the Hamiltonian $H_{\rm int}$ to the diagonal matrix $\Delta E(t)$ defined in eq. (5.28),

$$H_{\rm int} = U(t)\Delta E(t)U^\dagger(t) \ . \tag{7.2}$$

Then the equation for $\tilde{\rho}_N$ is

$$i\frac{d\tilde{\rho}_N}{dt} = [\tilde{H}, \tilde{\rho}_N] - \frac{i}{2}\{\Gamma_{NU}, \tilde{\rho}_N - \rho^{\rm eq}\} + i\mu_\alpha\tilde{\Gamma}^\alpha_{NU} \, , \tag{7.3}$$

where

$$\begin{aligned} \tilde{H} &= \frac{1}{2i}E^\dagger\left(U^\dagger\dot{U} - \dot{U}^\dagger U\right)E \, , \\ \Gamma_{NU} &= E^\dagger U^\dagger \Gamma_N U E \, , \\ \tilde{\Gamma}^\alpha_{NU} &= E^\dagger U^\dagger \tilde{\Gamma}^\alpha_N U E \ . \end{aligned} \tag{7.4}$$

Exactly the same procedure applies for the equation describing the antiparticles.

As was explained in [2], the set of equations (4.3), (4.4), (4.5) can be solved perturbatively for the case when all damping rates (symbolically Γ) are small enough, $\Gamma t \ll 1$. This is done in the following way: rewrite the differential equations (7.3) in the integral way, e.g.

$$\tilde{\rho}_N = -i\int_0^t dt' (\text{right hand side of eq. (7.3)}) \tag{7.5}$$

and then solve them iteratively. Then asymmetries in leptonic numbers (chemical potentials μ_α) are given by

$$\begin{aligned} \mu_\alpha = &\int_0^t dt' \int_0^{t'} dt'' {\rm Tr}\left[\left(\tilde{\Gamma}^\alpha_L(t')V(t',t'')\Gamma_N(t'')\right)V^\dagger(t',t'')\right] \\ &- \int_0^t dt' \int_0^{t'} dt'' {\rm Tr}\left[\left(\tilde{\Gamma}^{\alpha *}_L(t')\bar{V}(t',t'')\Gamma^*_N(t'')\right)\bar{V}^\dagger(t',t'')\right] \, , \end{aligned} \tag{7.6}$$

where

$$V(t',t'') = U(t')E(t')E^\dagger(t'')U^\dagger(t'')\,,$$
$$\bar{V}(t',t'') = U(t')^* E(t')E^\dagger(t'')U^T(t'') \tag{7.7}$$

(T corresponds to the transposed matrix). Equation (7.6) can be simplified,

$$\mu_\alpha(t) = 4\int_0^t dt' \int_0^{t'} dt''\ \ \mathrm{Im}\left[(U^\dagger(t')\tilde{\Gamma}^\alpha_L(t')U(t'))_{12}(U^\dagger(t'')\Gamma_N(t'')U(t''))_{21}\right]\times$$
$$\mathrm{Im}\left[\exp\left(i\int_{t''}^{t'} dt'''(E_2(t''') - E_3(t'''))\right)\right]\ . \tag{7.8}$$

As usual, the asymmetry contains a product of two imaginary parts. The first multiplier in (7.8) is associated with the CP-breaking complex phases in the Yukawa couplings, whereas the second corresponds to the oscillations between two singlet flavours. As was shown in [2], in the second order of perturbation theory and neglecting mass corrections $\mathcal{O}\left(\frac{M^2}{T^2}\right)$, the total leptonic asymmetry is zero, $\sum\mu_\alpha = 0$. It appears in the third order only, leading to an extra suppression of the order of Γt. In this work we will not go beyond the second order of perturbation theory, and account for extra suppression by multiplying the results by Γt. Note that the resonant production of dark matter sterile neutrinos occurs even if total lepton asymmetry is zero but individual flavour asymmetries are large enough [16, 2].

What happens for large times? For definiteness, suppose that the smallest damping rate for CP-even deviations from thermal equilibrium is Γ_{33}. Then, for $\Gamma_{33}t \gg 1$, the CP-even fluctuations thermalize, $\delta\rho_+ \ll 1$, and the source terms in (4.8) completely disappear, meaning that the production of lepton asymmetry stops.

In fact, an even stronger statement is true, namely that there is no generation of lepton asymmetry for $\Gamma_{22}t \gtrsim 1$. Indeed, in this regime all but one element of $\delta\rho_+$ are exponentially damped: the oscillatory off-diagonal part of the CP-even density matrix disappears at $(\Gamma_{22}+\Gamma_{33})t/2 \gtrsim 1$, and one of the diagonal elements at $\Gamma_{22}t \gtrsim 1$. So, in the mass basis

$$\delta\rho_+ = \begin{pmatrix} 0 & 0 \\ 0 & \delta\rho_+^{33} \end{pmatrix}\ . \tag{7.9}$$

For this type of deviation from thermal equilibrium the source terms in the equation for chemical potentials μ_α, eq. (4.10) vanish, $S_\alpha = 0$. The same is true for diagonal elements of the density matrix $\delta\rho_-$, accounting for asymmetries in singlet fermions. In other words, the leptogenesis ceases to work when coherence in oscillations of singlet fermions is lost, which happens when one of them is thermalized. The same conclusion is reached if the damping of coherent oscillations is inserted "by hands" into eq. (7.8).

To conclude, we expect that the asymmetry is maximal at $t_{\rm coh} \sim 2/(\Gamma_1+\Gamma_2)$. For $t > t_{\rm coh}$ the production of the asymmetry is switched off, and the asymmetries in different quantum numbers decay with the rates found in section 6.

Let us estimate the maximal possible asymmetry which can be created at $t \sim t_{\rm coh}$, corresponding to the temperature at which N_2 equilibrates, $T \simeq T_+$.

In the **Scenario III** for the singlet fermion mass differences the matrix $U(t)$ depends on time slowly. Indeed, when the tree level mass difference is much larger than the Higgs induced mass, the matrix $U(t)$ corresponds to the rotation by $\pi/4$, and $\dot{U} \sim \Delta m_\nu/\Delta\lambda$. Therefore, the asymmetries at time t are of the order of

$$\mu_\alpha(t) \simeq \delta_{\rm CP} \frac{F^4}{F_0^2} \Phi(t), \tag{7.10}$$

where

$$\Phi(t) = \int_0^t dt' \int_0^{t'} dt'' R(T', M) R(T'', M) \times {\rm Im} \left[\exp\left(i \int_{t''}^{t'} dt''' (E_2(t''') - E_3(t''')) \right) \right], \tag{7.11}$$

where the temperatures T', T'' correspond to the times t', t'', and $\delta_{\rm CP}$ is defined in (3.19). For $\epsilon \sim 1$, $\delta_{\rm CP}$ can be of the order of 1.

In the **Scenario I** for $T > T_\beta$ the mass difference is determined by the vev of the Higgs field only. Therefore, the temperature dependence of the matrix $U(t)$ can also be factored out up to mass corrections M^2/T^2, so that $\dot{U} \sim M^2/T^2$. However, the asymmetry in μ_α is suppressed in comparison with eq. (7.10) by a factor (at small ϵ)

$$S_I \simeq \left(\frac{2|h^\dagger h|_{23}}{|h^\dagger h|_{22}} \right)^2 \simeq \left(\frac{2\epsilon \Delta m_\nu}{\kappa m_{\rm atm}} \right)^2, \tag{7.12}$$

since in the limit $\epsilon \to 0$ the matrices $H_{\rm int}$ and Γ_N can be simultaneously diagonalized (cf. eqs. (5.22) and (5.2)), so that off-diagonal elements appearing in (7.8) are suppressed either by a factor S_I or by a mass to temperature ratio M^2/T^2. A similar factor appears in the **Scenario II** for $\epsilon \ll 1$.

For the generic case of **Scenarios II** the phase factor cannot be factored out and the equations are more complicated. We expect, however, that the discussion below has a general character, at least on the qualitative level.

It is instructive to find the behaviour of $\Phi(t)$ in different limits. For this end we will assume that the rate $R(T, M)$ can be approximately represented as $R(T, M) = AT^{-n}$, where n is some number. For example, for temperatures above the peak of production of singlet fermions $n \simeq 4$, at $T > 100\,{\rm GeV}$ $n \simeq 1$, whereas at temperatures below the peak $n \simeq -5$. The exponential in (7.10) can be written as

$$\int_{t''}^{t'} dt''' (E_2(t''') - E_3(t''')) = x(T') - x(T''), \tag{7.13}$$

where

$$x(T) = \int_0^t dt \left\langle \frac{M \delta M(T)}{p} \right\rangle \simeq 0.15 \frac{M \delta M(T) M_0}{T^3}, \tag{7.14}$$

and $\langle \ldots \rangle$ is the thermal average. The physical meaning of the parameter $x(T)$ is that $x(T)/2\pi$ gives the number of oscillations between singlet fermions from the end of inflation till the temperature T. Then one easily finds:

$$\Phi(t) = {\rm const} \int_0^{x(T)} dz_1 z_1^{(n-1)/3} \int_0^{z_1} dz_2 \sin(z_1 - z_2) z_2^{(n-1)/3}$$

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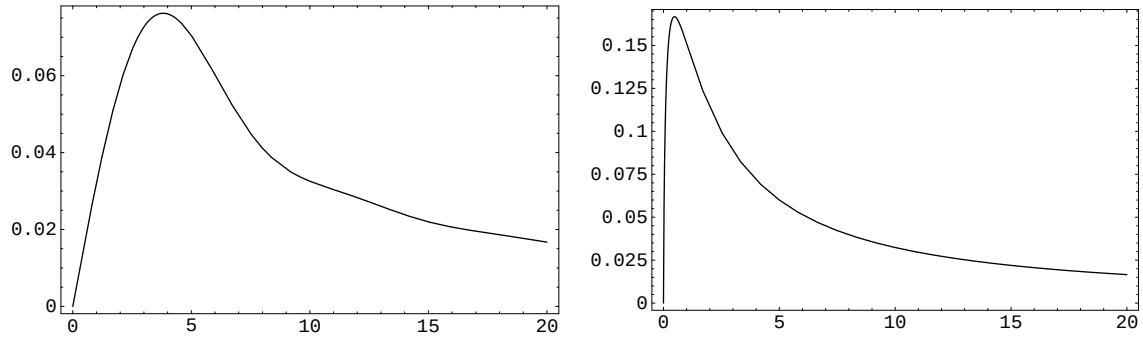

Figure 8: The behaviour of functions $F_+(x)$, $n = 4$ (left) and $F_-(x)$, $n = -5$ (right) counting the number of singlet fermion oscillations near the temperatures T_+ and T_-.

$$= \left(\frac{R(T)}{3H}\right)^2 F_+(x(T))\,, \tag{7.15}$$

where $F_+(x)$ in limiting cases is given by

$$F_+(x) = \begin{cases} \frac{27}{(n+2)(n+5)(2n+7)} x\ , & x \ll 1 \\ \frac{3}{2n+1}\frac{1}{x} \quad , & x \gg 1 \end{cases} \tag{7.16}$$

which is valid for $n > -1$, true for any temperatures $T > T_{\max}$, where $T_{\max}$ is the temperature at which the rate of N production is maximal. The plot of the function $F_+(x)$ for $n = 4$ is shown in figure 8.

Due to the very steep dependence of $\Phi(t)$ on the temperature the baryon asymmetry, produced at $T \simeq T_{\rm EW}$ can be much smaller than the lepton asymmetry, created at $T \simeq T_+$. Indeed, for $n = 4$ and for $x > 1$ one gets that $\mu_\alpha \propto 1/T^{15}$, so that a drop of the temperature by just a factor of 2, increases the asymmetry by a factor of 3×10^4. Including an extra factor $\Gamma t \simeq R(T)/3H$, accounting for the fact that baryon asymmetry is produced in third order of perturbation theory [2] amplify the difference even further.

Let us estimate the maximal possible asymmetry which can be produced at T_+. For this end suppose that the number of oscillations maximizes the function F_+ ($F_+^{\max} \simeq 0.076$ at $x \simeq 3.8$) and that CP-violation is maximal. Clearly, Δ cannot be larger than $\Delta_{\max} = 4/(9 \times 2 + 4) = 2/11$, where 4 is the total number of spin-states of $N_{2,3}$ and 9 is the number of spin-states of three leptonic generations. Thus,

$$\Delta \simeq \Delta_{\max} \frac{\epsilon F_+(x(T_+))}{F_+^{\max}}\,, \tag{7.17}$$

where the factor ϵ accounts for the fact that CP-violation goes away in the limit $\epsilon \to 0$.

Similar estimates apply for asymmetries in the other quantum numbers defined in section 6:

$$\delta Q_4 \simeq \Delta_{\max} \frac{\epsilon^2 F_+(x(T_+))}{F_+^{\max}}\ . \tag{7.18}$$

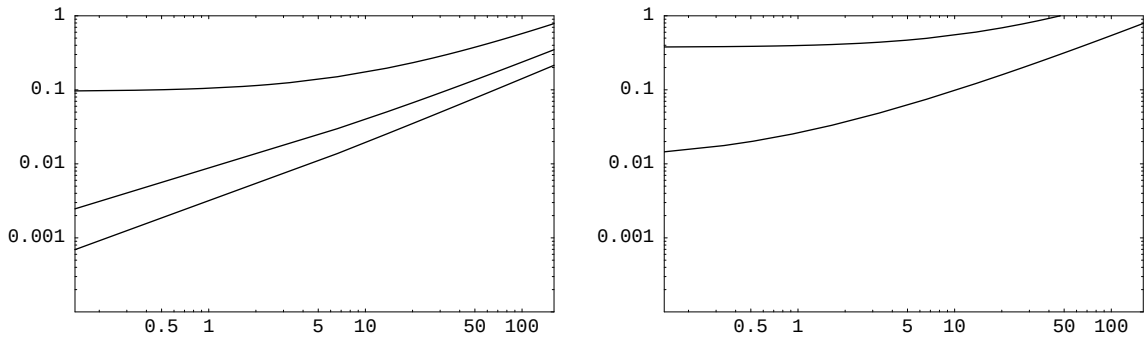

Figure 9: The region of the parameter space in [ϵ (vertical axis), M/GeV] plane in which the asymmetry defined in eq. (7.17), and then reduced due to damping, can be consistent with observations. The lower line corresponds to asymmetry $\Delta = 6.6 \times 10^{-9}$ (corresponding to observed baryonic asymmetry), the middle one to $\Delta = 6.6 \times 10^{-6}$ and the upper line in left panel to $\Delta = 6.6 \times 10^{-3}$. Left panel - normal hierarchy; right panel - inverted hierarchy.

An extra factor ϵ appears since the rate of creation or destruction of Q_4 is suppressed by ϵ^2 in comparison with the rates changing μ_α. As for the asymmetry in Q_5, one gets

$$\delta Q_5 \simeq \Delta_{\max} \frac{R_M(T_+, M)}{R(T_+, M)} \frac{\epsilon F_+(x(T_+))}{F_+^{\max}} , \tag{7.19}$$

where the second factor takes into account that the processes with the change of Q_5 are suppressed in comparison with $L \leftrightarrow N_2$ transitions.

The asymmetries in different quantum numbers generated at $T \sim T_+$ are reduced later with the rates determined in section 6.

7.1 Constraints on singlet fermions from baryon asymmetry

The estimates of the leptonic asymmetry presented above allow to find constraints on the masses and couplings of the singlet fermions from the requirement that the produced lepton asymmetry is large enough to make baryon asymmetry at the freezing point of sphaleron processes.

Consider first **Scenarios I, II** for the singlet fermion mass difference. If $T_+ > T_{\rm EW}$, the asymmetry generation in this case occurs in the resonant regime as the number of oscillations at temperature T_+ does not depend on parameters and is of the order of one,

$$x(T_+) \simeq 12 \frac{v^2(T)}{v^2} .$$

We present in figure 9 the region of the parameter space in which the baryon asymmetry (7.17), damped by a factor $\exp(-S_+(T_{\rm EW}))$ can exceed the observed value for normal and inverted hierarchies. We take the sphaleron freeze-out temperature to be 175 GeV, corresponding to the Higgs mass 200 GeV [62] and account for a suppression factor S_I defined in eq. (7.12).

The asymmetry related to the charge Q_4, eq. (7.18) can exceed the observed baryon asymmetry for $\epsilon \gtrsim 10^{-4}$ for normal hierarchy and for $\epsilon \gtrsim 10^{-2}$ for a wide range of the

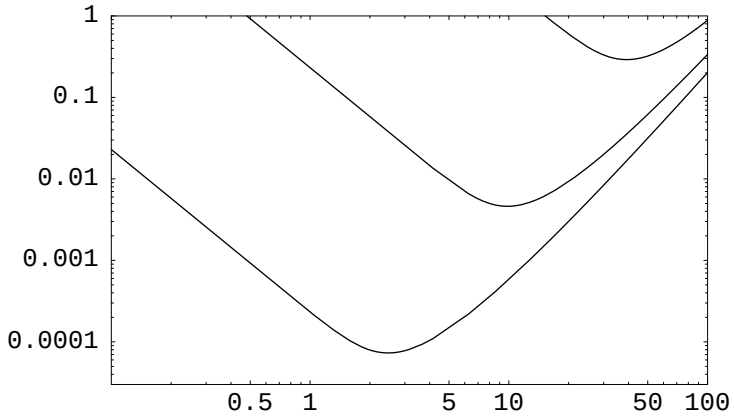

Figure 10: The region of the parameter space in [ϵ (vertical axis), M/GeV] plane in which the asymmetry defined in eq. (7.19) and reduced later due to damping discussed in section 6, can be consistent with observations. The upper line corresponds to asymmetry $\Delta = 6.6 \times 10^{-3}$, the middle one to $\Delta = 6.6 \times 10^{-6}$ and the lower line to $\Delta = 6.6 \times 10^{-9}$. We took the normal hierarchy case.

singlet fermion masses, including $M > M_W$. The fact that the baryon asymmetry generation is also possible for masses so large was missed in [2] and is due to the fact that the charge Q_4 is protected from erasure for small ϵ, whatever the value of M is.

In figure 10 we show the region of the parameter space where the asymmetry in Q_5 can exceed the observed value for the case of the normal hierarchy. The parameter ϵ is bounded from below by $\epsilon \simeq 7 \times 10^{-5}$, and the mass from above by $M \simeq 100\,\mathrm{GeV}$. These results refine the estimates presented in [7].

In the **Scenario III** the leptogenesis goes off the resonance and the available parameter space decreases. Comparing eq. (7.17) with observed baryon asymmetry one can put an upper bound on the mass difference difference of singlet fermions, $\delta M/M < 4 \times 10^{-8} \kappa^3 (M/\mathrm{GeV})$, valid if $T_+ < T_{\mathrm{EW}}$, $M \lesssim 50\,\mathrm{GeV}$ and $\epsilon \sim 1$. If T_+ lies in the symmetric phase of the electroweak theory, $T_+ \gtrsim 250\,\mathrm{GeV}$, a constraint from [2], $\delta M/M < 6 \times 10^{-8} (M/\mathrm{GeV})^{\frac{5}{2}}$ should be used.

7.2 Low temperature lepton asymmetry

Let us find now the region of parameters which can lead potentially to the generation of a large lepton asymmetry ($\Delta L/L > 2 \times 10^{-3}$, as required by observational constraints, discussed in [49]). Clearly, the constraints coming from baryon asymmetry are much weaker than those related to the large lepton asymmetry at lower temperatures. As we have already discussed, the asymmetry can be generated somewhat above T_+ or below T_-.

We start from $T \simeq T_+$. Out of five different leptonic numbers discussed in section 6 only two can survive the subsequent evolution. These are the asymmetry in Q_4 in the **Scenario IIa**, provided ϵ is small enough and in Q_5 which is protected if the mass of singlet fermion is small enough, see figure 7.

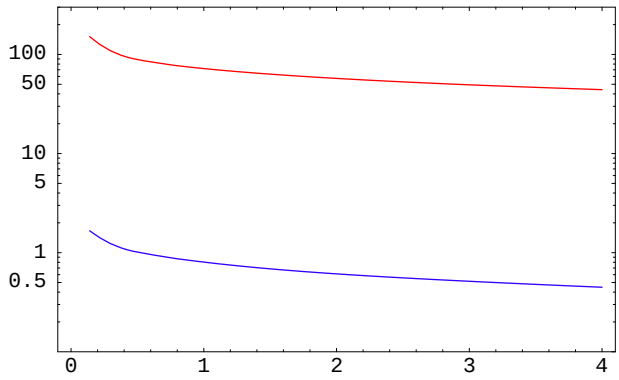

Figure 11: The number of oscillations (vertical axis) of singlet fermions at temperature T_+ for the Scenario I as a function of the fermion mass (in GeV) for normal (upper red curve) and inverted (lower blue curve) hierarchies. We took $\epsilon = 1$.

As we saw the number of oscillations at T_+ plays an essential role in the determination of the asymmetry. So, we present in figure 11 the quantity $x(T_+)$ for the **Scenario I** of singlet fermion mass difference for $\epsilon = 1$ for the case of normal and inverted hierarchies. For the generic choice of parameters for the **Scenario II** the number of oscillations is of the same order. However, by tuning the Majorana mass difference to the Higgs induced mass difference it can be made much smaller (for $\epsilon = 1$, see eq. (5.4)) than the numbers appearing in figure 11. For the **Scenario III** the number of oscillations is much larger than that in the **Scenario I** (by a factor $\delta M/m_{\rm atm}$ if the comparison is with normal hierarchy case).

Consider now the lepton asymmetry in Scenarios I-III.

Scenario I. The only possibility is to have an asymmetry in Q_5. Inserting different rates in (7.19) we get for $\epsilon \simeq 1$ the asymmetries plotted in figure 12 (assuming that the number of oscillations maximizes the asymmetry). For the normal hierarchy the asymmetry does not exceed 2×10^{-4} and thus is smaller than the minimal required number (2×10^{-3}) at least by a factor of 10. For the inverted hierarchy the maximal asymmetry is about 1×10^{-4}, a factor of 20 smaller than required. Though there are no orders of magnitude differences between potentially produced asymmetries and the required one, the conclusion that **Scenario I** cannot lead to necessary lepton asymmetry is robust. Indeed, in all estimates the CP-violating affects were assumed to be maximal, and other uncertainties were pushed in the direction which can only increase the asymmetry (for example, accounting for the number of oscillations will reduce the asymmetry for the case of normal hierarchy by a factor of 20).

In the **Scenario II** for generic choice of parameters the results for Q_5 stay the same as in the previous case. In other words, no sufficient asymmetry in Q_5 can be produced at T_+ for this case. Potentially, in the **Scenario IIa** the leptonic charge Q_4 can survive. However, this can only happen if $\epsilon < 2.4 \times 10^{-2}$. For ϵ so small the maximal asymmetry in Q_4 cannot exceed $\Delta_{\rm max}\epsilon^2 \simeq 10^{-4}$, too small to have any effect on dark matter production. Now, if the **Scenario III** for singlet fermion mass difference is realized, the asymmetry

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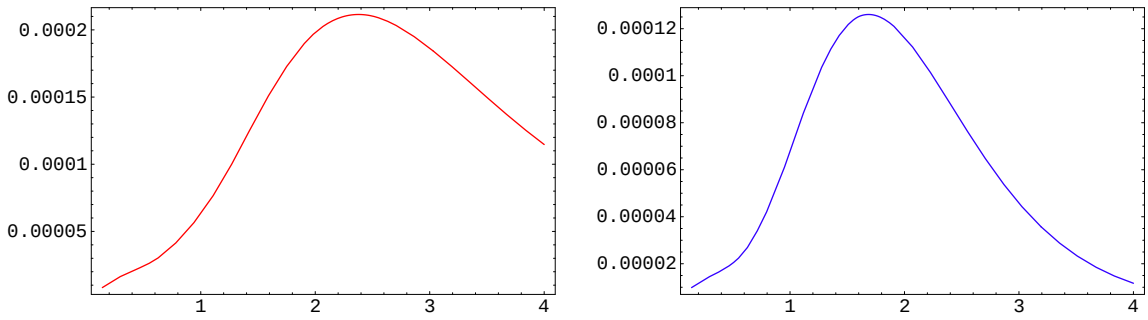

Figure 12: Maximal possible lepton asymmetry (vertical axis) generated at T_+ and survived till T_- for the Scenario I for normal (left) and inverted (right) hierarchies as a function of singlet lepton mass (in GeV). We took $\epsilon = 1$.

gets reduced by a factor $m_{\rm atm}/\delta M \ll 1$ in comparison with **Scenario I**. Since no large asymmetry can be produced in the **Scenario I**, **Scenario III** can be discarded as well.

To summarise, no generation of large lepton asymmetry at $T \simeq T_+$, which can survive till small temperatures, is possible.

Consider now a possibility of large lepton asymmetry generation at lower temperatures, $T \simeq T_-$. The oscillations of singlet fermions re-enter into coherence regime at $T \simeq T_-$, corresponding to t_-. Then, one can simply change the region of integration in (7.8):

$$\int_0^t dt' \int_0^{t'} dt'' \to \int_{t_-}^t dt' \int_{t_-}^{t'} dt'' \tag{7.20}$$

accounting for the fact that at $t < t_-$ the oscillations were exponentially damped. Correspondingly, the limits of integration in the phase factor $\Phi(T)$ defined by (7.11) must be changed. We get:

$$\begin{aligned} \Phi(t) &= \text{const} \times \text{Im} \int_{x(T)}^{\infty} dz_1 e^{iz_1} z_1^{(n-1)/3} \int_{x(T)}^{z_1} dz_2 e^{-iz_2} z_2^{(n-1)/3} \\ &= \left(\frac{R(T)}{3H}\right)^2 F_-(x(T))\,, \end{aligned} \tag{7.21}$$

where the plot of the function $F_-(x)$ for $n = -5$ is shown in figure 8. It reaches the maximal value $F_-^{\max} = 0.167$ at $x = 0.47$.

In limiting cases the function $F_-(x)$ is given by

$$F_-(x) = \begin{cases} -\frac{27}{(n+2)(n+5)(2n+7)} x\,, & x \ll 1 \\ -\frac{3}{2n+1}\frac{1}{x}\,, & x \gg 1 \end{cases} \tag{7.22}$$

which is valid for $n < -5$. The case of $n = -5$ requires a special treatment, leading to the asymptotic value $F_-(x) = -x\log(e\gamma x)$ for $x \ll 1$, where $e = 2.718\ldots$ and $\gamma = 0.577\ldots$ is the Euler constant.

To estimate the leptonic asymmetry generated at this time one can write

$$\Delta \simeq \Delta_{\max} \frac{\epsilon F_-(x(T_-))}{F_-^{\max}} \frac{\delta n}{n_\nu} , \tag{7.23}$$

where the factor $\delta n/n_\nu$ accounts for deviation of the sterile neutrino concentration from the equilibrium one at temperatures close to but below T_-. If $N_{2,3}$ decouple from the plasma being relativistic, $M \lesssim T_-$, the deviation of their concentration from equilibrium is suppressed by the factor

$$\delta n/n_\nu \simeq |n_{\rm eq}(T, M)/n_{\rm eq}(T, 0) - 1| \simeq 0.2 M^2/T_-^2 . \tag{7.24}$$

If the decoupling occurs when $T_- < M$, the corresponding factor is

$$\delta n/n_\nu \simeq 0.7 \left(\frac{M}{T_-}\right)^{3/2} \exp\left(-\frac{M}{T_-}\right) . \tag{7.25}$$

Since all reactions which change different leptonic numbers are out of equilibrium at temperatures below T_-, the asymmetries (7.23) stay intact.

Let us estimate the number of oscillations at $t \sim t_-$. Suppose first that $T_- \gg M$ so that the high temperature approximation can be used. Then with the use of eq. (5.27) the temperature T_- is given by

$$T_- \simeq \left(\frac{\epsilon M}{\kappa B G_F^2 m_{\rm atm} M_0}\right)^{1/3} \tag{7.26}$$

leading to

$$x(t_-) \simeq \frac{0.15 \kappa B}{\epsilon} (G_F M_0)^2 m_{\rm atm} \delta M . \tag{7.27}$$

The asymmetry is maximal if the number of oscillations is minimal. So, to get the maximal asymmetry we should take $\epsilon = 1$ and the minimal δM. For the **Scenario I** this corresponds to the inverted hierarchy of neutrino masses and to $x(T_-) \simeq 3.6 \times 10^3$. So, the asymmetry cannot exceed 10^{-4}, with the actual number being smaller as one has to account for the factor $\delta n/n_\nu < 1$ and for extra suppression from CP-breaking phases. For the normal hierarchy of neutrino masses the number of oscillations is larger by a factor of ~ 50, and for the **Scenario III** for the singlet fermion mass difference it is even higher. We conclude, therefore, that large lepton asymmetry, interesting for dark matter production, cannot be generated at $T \simeq T_-$ for **Scenarios I** and **III**, at least if $M \ll T_-$.

Let us find the critical singlet fermion mass where the relativistic approximation used above is not valid. Since the typical momentum of a fermion in the plasma is $\langle p \rangle \sim 3T$, we require $3T \simeq M$ and find that the singlet fermions decouple being non-relativistic if

$$M > M_{\rm crit} \simeq \left(\frac{27\epsilon}{\kappa B G_F^2 m_{\rm atm} M_0}\right)^{1/2} , \tag{7.28}$$

giving $M_{\rm crit} \simeq 30\,{\rm GeV}$ for $\epsilon = 1$. We will demonstrate now that the lepton asymmetry is also very small if the singlet leptons decouple in the non-relativistic regime (again **Scenarios I** and **III** are considered).

At large singlet fermion masses one can neglect the influence of the medium and consider the processes involving $N_{2,3}$ as if they were in the vacuum. The fastest reactions at temperatures $T < M_W$ are the decays $Z \to \nu N$ and $W \to lN$ (with the rate Γ_V), and decays or inverse decays of N to all possible leptonic or semi-leptonic channels (rate Γ_N). The rates of inverse W and Z decays, responsible for thermalisation, can be approximated as (at $M < M_W$)

$$\Gamma_V \simeq \frac{1}{3}\theta_0^2 n_V \left[\Gamma_{W\to l\nu} \exp\left(-\frac{M_W}{T}\right)\left(1-\frac{M^2}{M_W^2}\right)^{3/2} + 2\Gamma_{Z\to\bar{\nu}\nu}\exp\left(-\frac{M_Z}{T_L}\right)\left(1-\frac{M^2}{M_Z^2}\right)^{3/2}\right], \tag{7.29}$$

where $\Gamma_{W\to l\nu} \simeq 0.7\,\mathrm{GeV}$ and $\Gamma_{Z\to\bar{\nu}\nu} \simeq 0.5\,\mathrm{GeV}$ are the widths of the intermediate vector bosons, $n_V = 3$, and $\theta_0^2 \simeq \kappa m_{\mathrm{atm}}/(2\epsilon M)$.

The rate of inverse decays of N is of the order

$$\Gamma_N = A\frac{G_F^2 M^5 \theta_0^2}{192\pi^3}\left(1-\frac{M^2}{M_W^2}\right)^{-2}\exp\left(-\frac{M}{T}\right), \tag{7.30}$$

where A is proportional to the number of open channels for N_2 decays, $A \sim 10$ if $M > 10\,\mathrm{GeV}$ [11]. The temperature at which the oscillations of N start to be coherent can be determined from the condition $H = \Gamma_N + \Gamma_V$, and the lepton asymmetry from the relations (7.23), (7.25). The results for the temperature T_-, the number of oscillations and the lepton asymmetry are shown in figure 13. Note that for the non-relativistic case the number of oscillations is given by

$$x \simeq \frac{M_0 \delta M}{T^2}\ . \tag{7.31}$$

One can see that the asymmetry never exceeds 2×10^{-5} (inverted hierarchy) and 4×10^{-7} (normal hierarchy), which is well below the threshold for the resonant production of dark matter. We conclude, therefore, that no substantial asymmetry generation can occur after singlet fermions decouple in **Scenarios I** and **III**. The same conclusion is valid for the **Scenario II** for a generic choice of parameters.

On the other hand, for a special case of **Scenario IIa**, when the sterile fermion mass difference is much smaller than the active neutrino mass difference, the asymmetry production enters into resonance and the generation of large lepton asymmetries $\Delta L/L \gtrsim 2\times10^{-3}$ at T_- becomes possible for a variety of masses and couplings of singlet fermions. With the use of eqs. (5.7), (7.14), (7.26) one finds that if $\delta M(0) = 0$, the number of oscillations at $T = T_-$ does not depend on M and ϵ (for $\epsilon \ll 1$) and is given by

$$x(T_-) \simeq 0.15\left(\frac{p}{T}\right)^2 \frac{8}{\pi\alpha_W}\frac{7\pi^2}{360B}(2+\cos^2\theta_W) \simeq 10\ . \tag{7.32}$$

In other words, we are close to the resonance and a large asymmetry can be produced. In figure 14 we present the part of the parameter-space where the asymmetry may exceed the critical value.

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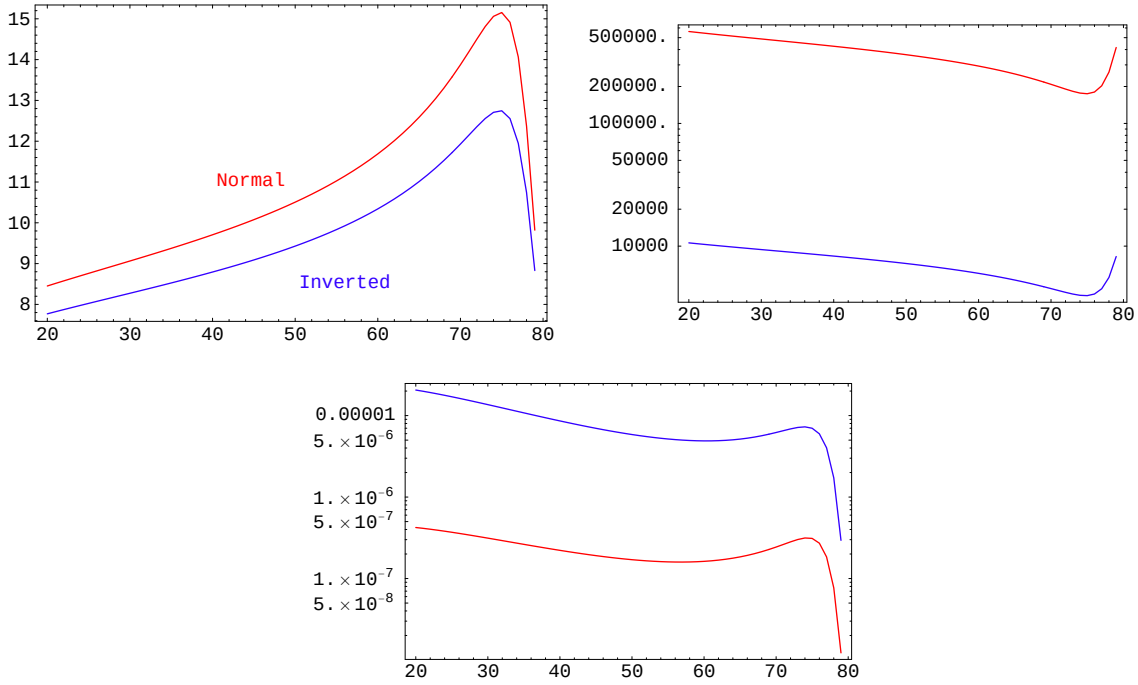

Figure 13: The temperature T_- in GeV (upper left), the number of oscillations (larger for normal hierarchy) of singlet fermions at temperature T_- (upper right) and the maximal lepton asymmetry (smaller for normal hierarchy) generated at T_- as a function of the fermion mass (in GeV) for normal (red curve) and inverted (blue curve) hierarchies. We took $\epsilon = 1$ and the **Scenario I**.

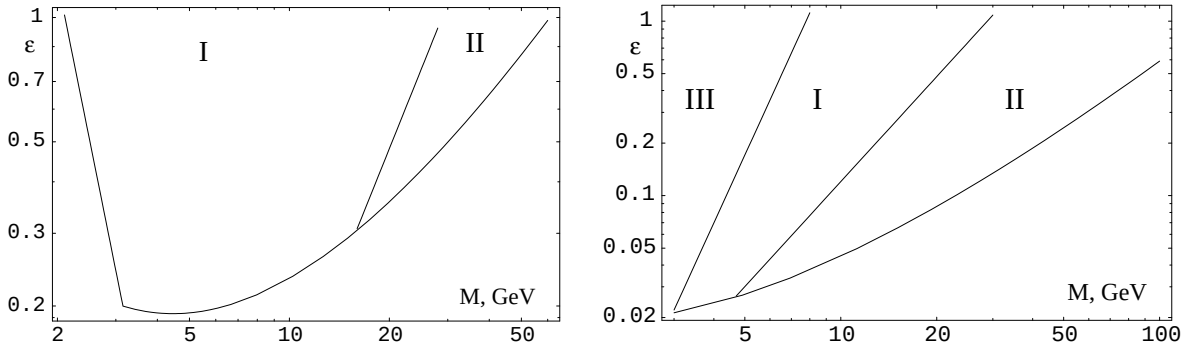

Figure 14: Left panel: the parameter-space (I and II) which can lead to the lepton asymmetry, produced at $T = T_-$ and exceeding 2×10^{-3}. Right panel: the parameter-space (I and II), which can lead to the lepton asymmetry, produced in decays of $N_{2,3}$ and exceeding 2×10^{-3}. In the region III $N_{2,3}$ decay below the temperature 100 MeV and thus do not contribute to resonant production of dark matter. In the region I (II) $N_{2,3}$ decouple being relativistic (non-relativistic).

In fact, yet another mechanism for late leptogenesis is possible in the **Scenario IIa** with the "tuned" mass difference. If $3T_- > M$, the singlet fermions decouple from the plasma being relativistic. Later, they decay with lepton number non-conservation and CP-

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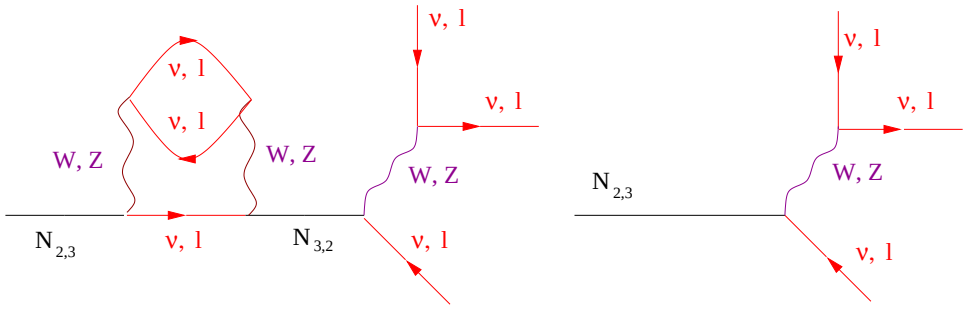

Figure 15: Diagrams for $N_{2,3}$ decay which can lead to large lepton asymmetry below the electroweak scale.

violation and, if they are degenerate enough, they will produce large lepton asymmetries.[8]

Let us estimate the value of the lepton asymmetry which can be created in decays of $N_{2,3}$. Since the Yukawa couplings are very small, the main contribution to CP asymmetry comes from the mixing between N_2 and N_3, as shown in figure 15. An estimate for asymmetry reads

$$\Delta \simeq \Delta_{\max} \left(\frac{\epsilon \Gamma_N}{\delta M}\right) \left[\frac{M^2}{\Gamma_N M_0}\right] , \tag{7.33}$$

where the the first term (ϵ) comes from CP-violation, the second term describes the resonance and is valid for $\delta M \gtrsim \Gamma_N$ (it should be replaced by 1 in the opposite limit), the third term accounts for equilibration of the asymmetry due to inverse N_2 decays. It should be replaced by 1 if $M^2 > \Gamma_N M_0$, i.e. for

$$\frac{M}{\mathrm{GeV}} < 19 \left(\frac{\epsilon}{2\times 10^{-3}}\right)^{1/2} \left(\frac{10}{\kappa A}\right)^{1/2} . \tag{7.34}$$

For the case $\delta M \sim \Gamma_N$ the asymmetry can be large and lead to the resonant production of dark matter sterile neutrino, provided $N_{2,3}$ decay above the temperature $\sim 100\,\mathrm{MeV}$, at which N_1 are created most effectively. The latter requirement leads to the constraint

$$\frac{M}{\mathrm{GeV}} > 1.4 \left(\frac{\epsilon}{2\times 10^{-3}}\right)^{1/4} \left(\frac{10}{\kappa A}\right)^{1/4} . \tag{7.35}$$

In figure 14 we present the part of the parameter-space where the asymmetry created in $N_{2,3}$ decays may exceed the critical value.

To constrain further the parameter-space of the model one should take into account the requirement that not only the low temperature lepton asymmetry must be large enough, but also that the baryon asymmetry is small. Given the number of CP-phases and other parameters we expect that anywhere in the regions shown in figure 14 the required hierarchy can be achieved by some choice of Yukawa couplings. However, for a generic case, in which

[8]The fact that CP-violation is greatly enhanced in the decays of degenerate particles is well known from K^0 physics. It was first suggested for baryogenesis in [63], discussed in [64] and studied in detail for TeV scale Majorana fermions in [65 – 67].

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no cancellation between different CP-violating phases takes place, the region of small singlet fermion masses and large ϵ is singled out.

Indeed, in the **Scenario IIa** the baryon asymmetry generation occurs in the resonant regime at $T_+ > T_{\rm EW}$, leading generally to large baryon asymmetries. The observed small baryon asymmetry can be derived moving out of the resonance, i.e. for $T_+ < T_{\rm EW}$. The number of oscillations at the electroweak temperature $T_{\rm EW} \simeq 175\,{\rm GeV}$ is (for $\epsilon \ll 1$):

$$x(T_{\rm EW}) \simeq 0.15 \frac{\kappa M_0 M m_{\rm atm} v^2(T_{\rm EW})}{4\epsilon T_{\rm EW}^3 v^2} \simeq \frac{0.12\kappa}{\epsilon}\left(\frac{M}{\rm GeV}\right) \tag{7.36}$$

and smaller than one if ϵ is large and M is small. In this regime the baryon asymmetry is suppressed by a factor

$$(R/3H)^3 \frac{x(T_{\rm EW})}{30 F_+^{\rm max}} \simeq 2 \times \left[\frac{0.02\kappa}{\epsilon}\left(\frac{M}{\rm GeV}\right)\right]^4 , \tag{7.37}$$

which is about 5×10^{-6} for $M \simeq 2\,{\rm GeV}$, $\epsilon \simeq 1\,, \kappa = 1$ (upper left corner in figure 14), producing roughly a correct hierarchy between high temperature baryon asymmetry and low temperature lepton asymmetry.

Finally, let us discuss the possibility that large lepton asymmetries $\Delta_0 \gtrsim 2 \times 10^{-3}$ were generated well above the electroweak temperature. Is it possible that they were not transferred to baryon asymmetry but survived till low temperatures?

As we have already found, the only leptonic numbers that can survive till low temperatures are related to the currents J^4_μ and J^5_μ, defined in (6.5), (6.6). Moreover, the only flavour structure of primordial asymmetry which is consistent with small baryon asymmetry is the one in which $L + \Delta N_2 = 0$, where L is a lepton number of active fermions, and ΔN_2 is the asymmetry in a more strongly interacting singlet fermion. Indeed, if L is large it will lead to large baryon asymmetry due to sphalerons. If ΔN_2 is large, a part of it will be transferred to L and then to baryon asymmetry. The amount of N_2 going to L is at least

$$\Delta_0 S_+(T_{\rm EW}) > 2 \times 10^{-4} \Delta_0 \gg \Delta B\,, \tag{7.38}$$

where we used the minimal possible rate $R(T, M)$ corresponding to $M \simeq m_\pi$ and $\epsilon = 1$. In other words, the only possibility is to have large asymmetry in N_3, $\Delta N_3 = \Delta_0$ and assume that $\epsilon \ll 1$, suppressing the transitions $N_3 \to L$.

Now, four different possibilities can be realised. If the reactions changing Q_4 and Q_5 were both in thermal equilibrium, no primordial asymmetry will survive. If, on the contrary, none of the reactions changing Q_4 and Q_5 were in thermal equilibrium, a large asymmetry in N_3 will not be transferred to an asymmetry in active leptons, and, therefore, no resonant production of dark matter sterile neutrinos is possible. So, to get large lepton asymmetry at low temperatures one must require that one of charges out of Q_4 and Q_5 must be conserved and the other equilibrate. In figure 16 we present the parameter-space in which the primordial asymmetry in N_3 induces a baryon asymmetry smaller than the observed one but leads to large low temperature lepton asymmetry. It requires rather small values of the parameter ϵ.

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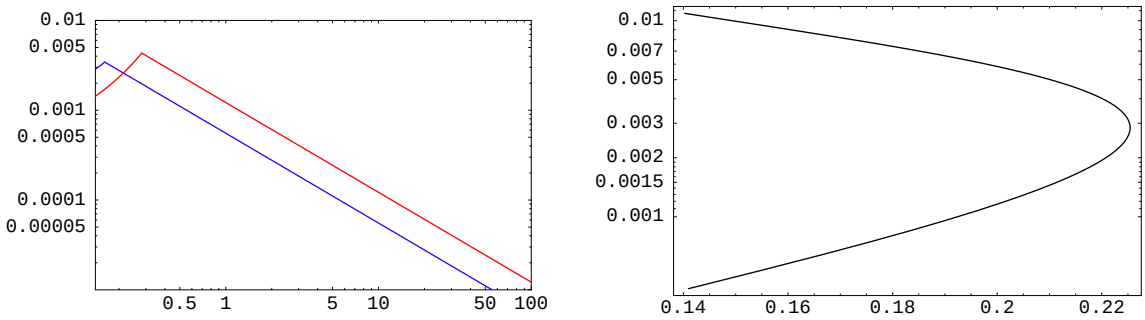

Figure 16: Left panel: Part of the parameter space corresponding to conservation of Q_4 and non-conservation of Q_5. This can only be realised in **Scenario IIa** in which $T_\beta < T_-$. Vertical axis: ϵ, horizontal axis: mass in GeV. The admitted regions are below the curves. Upper red line - normal hierarchy, lower blue line - inverted hierarchy. Right panel: Part of the parameter space corresponding to conservation of Q_5 and non-conservation of Q_4. It is required that $T_\beta > T_-$. Vertical axis: ϵ, horizontal axis: mass in GeV. The admitted region is to the left of the curve. No parameter space is allowed for the inverted hierarchy case.

8. Fine tunings or new symmetries?

The requirement that the νMSM produces both baryon asymmetry and dark matter in amounts required by observations puts very stringent constraints on the parameters of the model. In this section we will discuss whether these constraints, appearing as different fine-tunings in the Lagrangian of the νMSM, can indicate the existence of some hidden approximate symmetries. These symmetries, if exist, cannot be explained in the framework of the νMSM itself, as this model is based on a renormalizable field theory which may be valid all the way up to the Planck scale [12]. At the same time, their presence can give some hints on the properties of more fundamental theory, replacing the νMSM at high energies.

We start from the relative strength of Yukawa interactions of singlet fermions N_2 and N_3. A non-trivial constraint on the νMSM parameters is coming from the requirement that the baryon asymmetry at the electroweak scale must be much smaller than the lepton asymmetry at small temperatures. It tells that the parameter ϵ should be close to its maximal value, $\epsilon \sim 1$. For ϵ that large the lepton number U(1) symmetry, introduced in [7], is strongly broken in the singlet fermion Yukawa sector, but is respected by the Majorana masses of the singlet fermions and by charged lepton Yukawas. Therefore, one may wonder if some other global symmetry, respected both by the Yukawa couplings and by Majorana masses, may exist for the extreme case $\epsilon = 1$.

As was discussed in [7], such a symmetry does not exist if *both charged and singlet* lepton Yukawa couplings are taken into account. If, however, charged lepton Yukawas are disregarded, quite a symmetric singlet lepton interaction can be found in the *inverted* hierarchy case. Indeed, for the case $m_1 = m_2$, $\theta_{23} = \pi/4$, and $\theta_{13} = 0$ the fields L_2 and L_3 defined in (3.9) are the *orthogonal* mixtures of different leptonic flavours. Thus, for $\epsilon = 1$ the Yukawa part of Lagrangian (3.3) is symmetric with respect to the non-Abelian flavour

group SU(2) (broken, of course, by the charged lepton Yukawa couplings). This group is broken down to U(1) by the Majorana mass term $M\bar{N}_2^c N_3$. This U(1) group is then only slightly broken by the diagonal mass terms $\sim \Delta M_M \ll M$ and by corrections in the Yukawa sector, which can be as small as $\delta_{\rm inv} \sim 0.01$ defined in eq. (3.12). So, if the existence of slightly broken approximate symmetry indeed matters, then the inverted hierarchy of neutrino masses with small $\theta_{13} \sim \delta_{\rm inv}$ and small deviation of the angle $\delta\theta_{23} \sim \delta_{\rm inv}$ from the maximal value is preferred.

Interestingly, for $|\epsilon - 1| \sim \delta_{\rm inv}$ and $\delta_{\rm inv} \ll 1$ the interactions of the heavy neutral lepton mass eigenstates with intermediate weak vector bosons are universal and characterized by the same mixing angle

$$\theta_M^2 = \frac{F^2 v^2}{M^2} = \frac{m_{\rm atm}}{M} \ . \tag{8.1}$$

Also, both the high temperature baryogenesis and low temperature leptogenesis can take place.

Let us now try to guess what kind of couplings of the dark matter sterile neutrino with leptons may lead to some non-trivial symmetries. The phenomenology of DM sterile neutrino requires its mass be much smaller than the mass of the singlet fermions responsible for baryon asymmetry and that its Yukawa constants are much smaller than those for heavier neutral leptons. This leads to a conjecture that the singlet fermion Majorana masses could be proportional to their Yukawa couplings, satisfied already for $N_{2,3}$ in the construction presented above. If true, then the mixing angles of all three sterile leptons with neutrinos are the same, and the interaction of them with W and Z bosons exhibits the global SU(3) symmetry, which exists for charged leptons. If this hypothesis happens to be correct, the mixing angle of DM sterile neutrino is predicted to be

$$\theta_{DM}^2 = \frac{\sum_\alpha |h_{\alpha 1}|^2 v^2}{M_1^2} = \theta_M^2 = \frac{m_{\rm atm}}{M} \simeq 2.5 \times 10^{-11} \, , \tag{8.2}$$

corresponding to $M \simeq 2\,{\rm GeV}$, a preferred value leading to the required hierarchy between baryon asymmetry and low temperature lepton asymmetry. For this value of the mixing angle the mass of DM sterile neutrino is bounded from above by $M_1 \lesssim 8$ keV by X-ray observations (see the plots presented in ref. [49]). If the Lyman-α bounds of refs. [44, 45] are correct, then $M_1 \gtrsim 4$ keV (see the discussion in ref. [49]). To exclude or verify this prediction, the current X-ray constraints must be improved by a factor of 10.

Yet another fine-tuning which is necessary for creation of large low-temperature lepton asymmetry, is eq. (2.14), leading to **Scenario IIa** for singlet fermion mass difference. Though the origin of different parameters even in the Standard Model remains a mystery, it is tempting to speculate how this relation, equivalent to

$$2(h^\dagger h)_{23} v^2 + M(\Delta M_{22}^* + \Delta M_{33}) \simeq 0 \, , \tag{8.3}$$

may come from some more fundamental theory. In the νMSM described by Lagrangian (2.1) the first term in this condition is due to the Higgs condensate while the second is due to Majorana masses of singlet fermions and, therefore, they have completely different nature. Clearly, a correlation between two independent dimensionfull parameters

would be a miracle if the νMSM were the final fundamental theory. This is not so if the mass parameters in the νMSM have the common source, as in the model of [6], where the Higgs boson and the neutral fermion masses come from the vacuum expectation value of the nearly conformally coupled scalar field χ, singlet with respect to the SM gauge group. In this case the relation (8.3) turns into a connection between the Yukawa coupling constants in the sterile neutrino sector of the νMSM. In fact, all phenomenological and cosmological requirements to the parameters of the νMSM with extra scalar field χ can be encoded in a simple Lagrangian, kind of Effective Theory of Everything (ETOE), containing just few dimensionless parameters, their powers, and one mass scale. It has the form:

$$\mathcal{L}_{\nu\mathrm{MSM}} \to \mathcal{L}_{\nu\mathrm{MSM[M\to 0]}} + \frac{1}{2f_0 f_1}(\partial_\mu \chi)^2 - \frac{\chi}{2}\, \bar{N_I}^c m_{IJ} N_J + \mathrm{h.c.} - \mathrm{V}(\Phi,\chi) + \mathcal{L}_\mathrm{G}\,, \quad (8.4)$$

where the first term is the νMSM Lagrangian without Higgs potential and with all dimensionfull parameters (Higgs and Majorana masses) put to zero, the constants f_0 and f_1 will be specified below. The scalar potential is given by

$$V(\Phi,\chi) = \lambda\left(\Phi^\dagger\Phi - \chi^2\right)^2 + \beta(\chi^2 - v^2)^2\,, \quad (8.5)$$

where $\lambda \sim \beta \sim 1/10$ are the Higgs and χ self-couplings correspondingly, v is the Higgs vev. The gravity part is

$$\mathcal{L}_G = -\left(\frac{1}{f_0^6}\chi^2 + \frac{\lambda}{f_0}\Phi^\dagger\Phi\right)\frac{R}{2}\,, \quad (8.6)$$

where R is the scalar curvature. This is a Lagrangian of "induced gravity" going back to refs. [72, 73] (see also [6] in the νMSM context). The Yukawa couplings $h_{\alpha I}$ in eq. (2.1) are written as

$$h_{\alpha I} = f_0 f_{\alpha J} m_{JI}\,, \quad (8.7)$$

where $f_{\alpha J}$ is an arbitrary complex matrix with elements $f_{\alpha J} \sim 1$ and

$$m_{IJ} = f_1\left[\mathcal{M}_0 - \frac{1}{2}f_0^2\left(f^\dagger f\mathcal{M}_0 + \mathrm{transposed}\right)\right] \quad (8.8)$$

with

$$\mathcal{M}_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + f_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a \end{pmatrix}\,, \quad (8.9)$$

where $a \sim 1$ is a real number. The second term in (8.8) is chosen in such a way that eq. (8.3) is automatically satisfied for any choice of f_0, f_1 and $f_{\alpha J}$.

The parameter $f_0 = (v/M_P)^{\frac{1}{3}} \simeq 4\times 10^{-6}$, where $M_P = (8\pi G_N)^{-\frac{1}{2}} = 2.4\times 10^{18}\,\mathrm{GeV}$ is the reduced Planck scale, appears in several places in Lagrangian (8.4). It is fixed from the requirement to provide the known Newton constant and the correct phenomenology of singlet fermions, as we describe in what follows.

The parameter $f_1 \sim \delta_{\mathrm{inv}} \sim 10^{-2}$ sets the mass of the singlet fermions (giving baryon and lepton asymmetries) in the GeV region, $M_{2,3} \sim f_1 v$. The mass of dark matter neutrino $M_1 \sim f_0 M_{2,3}$ is then in $\mathcal{O}(10)$ keV region, masses of active neutrinos in the fraction of eV

region, $m_\nu \sim f_0^2 M_{2,3}$. In addition, f_0 makes the Yukawa coupling of sterile neutrino to be small compared with the Yukawa couplings of $N_{2,3}$ by a factor f_0, exactly what is needed to produce them in the early universe in right amounts to play the role of dark matter. Moreover, $\lambda/(2f_0) \sim 2\times 10^4$, appearing in the conformal coupling of the Higgs field to Ricci scalar R, leads to inflation producing correct amplitude of primordial fluctuations [13]. The field χ is very light due to its conformal coupling to gravity ($m_\chi \sim \sqrt{\beta} f_0^3 v \sim 10^{-5}\,\mathrm{eV}$), but practically decouples from the fields of the νMSM [74, 75] (see also [13]). In contrast with [6], where the terms (8.6) were not introduced, it plays no role in inflation and in production of dark matter sterile neutrinos.

The author has no idea from were the structures discussed above can be coming from but is amazed by some numerical coincidences they uncover.

9. Conclusions

In this work we scrutinized the mechanism of leptogenesis via oscillations of light singlet fermions and determined the parameter space of the νMSM which can lead to successful baryogenesis. The kinetic processes in the model are quite complicated as they are characterised by a number of different time scales and by fluctuations (deviations from thermal equilibrium) of different nature, interacting with each other.

The first sector includes *CP-even deviations* from thermal equilibrium in the system of almost degenerate singlet fermions. These fluctuations give a "source" term for baryogenesis; creation of lepton asymmetry switches off when these deviations are damped away. The kinetic evolution of these fluctuations is governed by four different time scales: two equilibration rates for $N_{2,3}$, the rate of losing of quantum coherence in oscillations of singlet fermions, and the rate of oscillations, related to the mass difference between singlet fermions. In the paper we estimated all these time scales (section 5). We found, in particular, the temperature dependence of the oscillation time, essential for **Scenarios I** and **II** for singlet fermion mass difference.

The second sector includes *CP-odd deviations* from thermal equilibrium in the system of singlet fermions and active leptons. There are 7 different essential kinetic time scales there. The first 4 are similar to those described above, three others govern the damping of asymmetries in different active leptonic flavours.

We established that the oscillations of $N_{2,3}$ must be *coherent* for effective leptogenesis. This is only true if *both* N_2 and N_3 are out of thermal equilibrium. In other words, lepton asymmetry increases in time till one of the singlet fermions, which interacts more strongly with the plasma (N_2 in our notations) enters in thermal equilibrium. After this moment the coherence in singlet fermion oscillations is lost, and asymmetries in different quantum numbers (which we identified) are damped with the rates, which we determined in section 6.

We found that baryogenesis may occur in a wide range of singlet lepton masses ranging from 140 MeV, allowed by experimental and BBN constraints, to the masses exceeding the electroweak scale. An essential requirement is a near degeneracy of a pair of the heavy neutral leptons. In addition, the parameter ϵ, characterising the breaking of the U(1) leptonic symmetry, cannot be smaller than 7×10^{-5}.

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We determined explicitly the CP-violating phase which drives baryogenesis in the model and demonstrated that it cannot be expressed only in terms of CP-violating phases of the active neutrino mixing matrix. Moreover, we found that the baryon asymmetry is non-zero in the limit of small θ_{13}.

We showed, furthermore, that the νMSM interactions of singlet fermions may produce a significant low temperature lepton asymmetry, being consistent with neutrino oscillation experiments and leading to the observed baryon asymmetry of the universe. In a companion paper [49], we show that this lepton asymmetry can account for all the dark matter in the universe. Thus, the νMSM *without introduction of any new physics or fields* such as the inflaton may happen to be a correct effective field theory all the way up the Planck scale [12] explaining a variety of phenomena that the SM fails to deal with. It is intriguing that the production of the baryon asymmetry of the universe and of the dark matter is due to essentially the same mechanism, making a step towards understanding why the abundances of dark and baryonic matters are roughly the same.

We also found that large lepton asymmetries in singlet fermions N_3, which could have been generated above the electroweak scale, may not be in conflict with the observed baryon asymmetry and can survive till low temperatures in a specific part of the νMSM parameter space. It corresponds to masses above 140 MeV and small $\epsilon < 5 \times 10^{-3}$ and also require **Scenario IIa** for the singlet fermion mass difference. Another possibility is to have singlet fermion masses near the pion mass and ϵ in the range $5 \times 10^{-4} < \epsilon < 0.01$. These regions can be explored in kaon experiments and in searches for singlet fermion decays [11].

The requirement that the νMSM produces a lepton asymmetry large enough to speed up the dark matter production allows to constrain considerably the parameters of the νMSM. The most non-trivial requirement is (2.14), telling that the *zero-temperature difference* between masses of the *physical* singlet fermions must be much smaller than the active neutrino mass differences. For this choice of parameters the baryon asymmetry is generated at temperatures close to the sphaleron freeze-out, $T \sim 130 - 175\,\mathrm{GeV}$, and a large lepton asymmetry at relatively small temperatures, $T = T_- \sim 0.1 \; - \; 10\,\mathrm{GeV}$, corresponding to the decoupling of singlet fermions from the plasma or to their decays. Later the lepton asymmetry is transferred to the dark matter population of sterile neutrinos. The asymmetry generation mechanism works for all singlet lepton masses admitted by experimental and BBN constraints discussed in [11] and for both types of neutrino mass hierarchies; to produce the low temperature lepton asymmetry required for resonant dark matter production the parameter ϵ should be large enough, $\epsilon \gtrsim 2 \times 10^{-2}$. Moreover, the requirement of having a much smaller baryon asymmetry favours large $\epsilon \sim 1$ and singlet fermion masses in the $\mathcal{O}(\mathrm{GeV})$ range. Particles with these properties can be searched for at existing accelerators [11], which is however very challenging due to the large value of ϵ, leading to a suppression of their production and to a decrease of their decay rates. At the same time, the CP-asymmetry in their decays must be at least on the level of few %.

We speculated on the origin of the necessary fine-tunings in the νMSM and proposed a Lagrangian, containing two dimensionless parameters and their powers, which encodes different relations required for the phenomenological success of the model. We found, in particular, that the theory with $\epsilon = 1$ and inverted hierarchy of neutrino masses exhibits

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a SU(2) flavour symmetry in the singlet fermion Yukawa sector, broken to U(1) by the Majorana mass term. The magnitude of the breaking of this U(1) group is small and is of the order $\frac{\Delta m^2_{\rm sol}}{4\Delta m^2_{\rm atm}} \simeq 8 \times 10^{-3}$. In a search of a "maximally symmetric" version of the νMSM we found that it is phenomenologically acceptable to think that the strength of the weak interactions of all types of singlet fermions is universal. This conjecture leads to a specific prediction for the mixing angle of dark matter sterile neutrino N_1, potentially testable with the help of *existing* X-ray satellites.

Finally, a word of warning. All the constraints discussed above are applicable only in the case when at temperatures well above the electroweak scale concentrations of all singlet leptons are zero. In particular, if the dark matter sterile neutrinos are generated above the electroweak scale in right amounts, no generation of large lepton asymmetry is needed below the electroweak scale.

Acknowledgments

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A. The rates of singlet fermions production

We specify in this appendix the ingredients that went into producing figure 3.

The basic formalism we follow is that of refs. [8, 9]. More precisely, the quantity Y in figure 3 is given by eq. (4.8) of ref. [9], while Y_M contains the additional weight $(q_0 - q)/(q_0 + q)$, cf. eq. (5.24).

The main difference with respect to the analysis of ref. [9] is that we now consider the heavy sterile neutrinos, and that the temperatures are correspondingly higher. This implies that the exponentially suppressed 1-loop corrections (eq. (3.1) of ref. [9]) start to dominate over the 2-loop terms (section 3.2 of ref. [9]). More precisely, the main changes to the numerical code are as follows:

- Because of the higher temperatures, the contribution of the bottom quark has been added to the 2-loop processes listed in table 1 of ref. [9].

- Once the temperature increases above 20 GeV or so, the treatment of 2-loop effects through the Fermi model is no longer justified. Therefore we smoothly switch off the 2-loop contributions within the range $T = (15 \ldots 30)\,\mathrm{GeV}$.

- Concerning the 1-loop effects, the graphs to be considered are given in figure 2 of the present paper (except that we use here a basis where the Majorana mass matrix is flavour-diagonal). In the top graph, the particle in the loop can either be a Higgs or a Goldstone. In addition, the self-energy of the active neutrino, appearing in the bottom graph, depends on the gauge choice (because the active neutrino is off-shell).

Now, the simplest gauge choice in this context is that of Feynman. Then the real part of the active neutrino self-energy (the function b) can be taken directly from ref. [68] and the imaginary part from eq. (3.1) of ref. [9]. At the same time, the top graph of figure 2 amounts to

$$\delta R(T,\mathbf{q}) = \frac{2n_{\rm F}(q^0)}{(2\pi)^3 2q^0}\sum_{\alpha=1}^{3}\frac{|M_D|^2_{\alpha I}}{m_W^2}{\rm Tr}\left[\not{Q}\, a_L\, {\rm Im}\,\not{\Sigma}_{\rm Higgs}\, a_R\right] , \tag{A.1}$$

where ${\rm Im}\,\not{\Sigma}_{\rm Higgs}$ has exactly the form in eq. (3.1) of ref. [9], with three channels characterized by $p_C = 1; m_C = m_H; m_{l_C} = m_{\nu_\alpha}$; $p_C = 1; m_C = m_Z; m_{l_C} = m_{\nu_\alpha}$; and $p_C = 2; m_C = m_W; m_{l_C} = m_{l_\alpha}$.

Another possible choice is the unitary gauge. Then the Goldstone contributions can be dropped from the top graph, but the active neutrino self-energy needs to be modified. We have checked that after the appropriate changes, the numerical results in the two gauges differ by an amount which is insignificant on our resolution.

- Once the temperature increases to several tens of GeV, the evolution of the Higgs vacuum expectation value needs to be taken into account. We do this by scaling $\sqrt{2}v(T) = 246\,{\rm GeV}\sqrt{1 - T^2/T_0^2}$, where T_0 is fixed through the knowledge that the sphaleron freeze-out temperature $T_{\rm EW}$, where we start our evolution, is characterized by $\sqrt{2}v(T_{\rm EW}) \simeq T_{\rm EW}$. We choose $m_H \simeq 200\,{\rm GeV}$ and then, according to ref. [62], $T_{\rm EW} \simeq 175\,{\rm GeV}$. All physical particle masses are rescaled by $v(T)/v(0)$.

Apart from these changes, the numerical techniques used are identical to those in ref. [9].

B. Lower bounds on Yukawa couplings

In this appendix we present a lower bound on the following combinations of Yukawa couplings which will appear in the analysis of equilibration in the early universe,

$$|f_{\alpha\alpha}|^2 \equiv \left(|h_{\alpha 2}|^2 + |h_{\alpha 3}|^2\right) \ . \tag{B.1}$$

With the use of (2.6) one can see that the minimal value of $|f_{\alpha\alpha}|^2$ is simply $|[M_\nu]_{\alpha\alpha}|M/v^2$. The smallest Yukawa couplings correspond to the smallest value of the Majorana neutrino mass, which we take to be $M \simeq m_\pi \simeq 140\,{\rm MeV}$ (the mass of the pion is introduced as a useful parametrisation) (smaller values would be in conflict with predictions of BBN [69, 70] and experiments devoted to the search of singlet fermions [71, 7, 11]). Inserting the central values for neutrino masses and mixing angles from [56]: $\Delta m^2_{\rm sol} = 8.0 \times 10^{-5}\,{\rm eV}^2$, $\Delta m^2_{\rm atm} = 2.5 \times 10^{-3}\,{\rm eV}^2$, $\theta_{23} = \pi/4$, $\tan^2(\theta_{12}) = 0.45$, $\theta_{13} = 0$, and choosing the unknown CP-violating phases in a way to minimize the Yukawa couplings, we get for the normal hierarchy:

$$|f_{ee}|^2 > 1.3 \times 10^{-17}\,, \quad |f_{\mu\mu}|^2 > 10^{-16}, \quad |f_{\tau\tau}|^2 > 10^{-16} \tag{B.2}$$

and for the inverted hierarchy

$$|f_{ee}|^2 > 8.8 \times 10^{-17}\,, \quad |f_{\mu\mu}|^2 > 4.4 \times 10^{-17}, \quad |f_{\tau\tau}|^2 > 4.4 \times 10^{-17}\ . \tag{B.3}$$

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These numbers change somewhat if the neutrino mixing parameters are varied in the experimentally admitted ranges. To get a minimal possible value of, say, $|f_{ee}|^2$ one should take the maximal possible atmospheric mass difference ($2.7 \times 10^{-3}\,\mathrm{eV}$), minimal solar mass difference ($7.7 \times 10^{-5}\,\mathrm{eV}$), minimal $\theta_{12} \simeq 0.56$ and maximal $\theta_{13} \simeq 0.11$, leading to

$$|f_{ee}|^2 > 8.4 \times 10^{-18} \ . \tag{B.4}$$

If $M > m_\pi$ then the lower bounds are stronger by a factor M/m_π.

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